# Competitive Routing on a Bounded-Degree Plane Spanner

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August 4, 2012

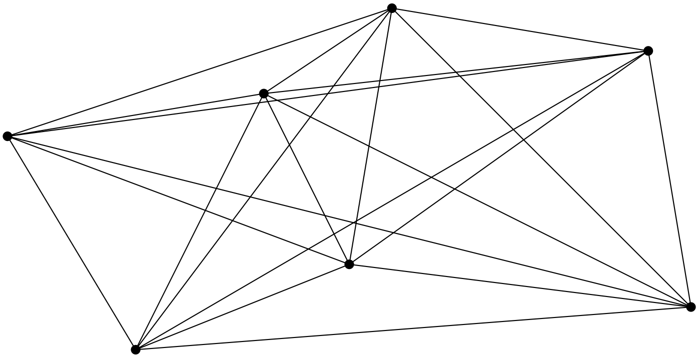
# Geometric Spanners

Given:

- Set of points in the plane

Goal:

- Approximate the complete Euclidean graph



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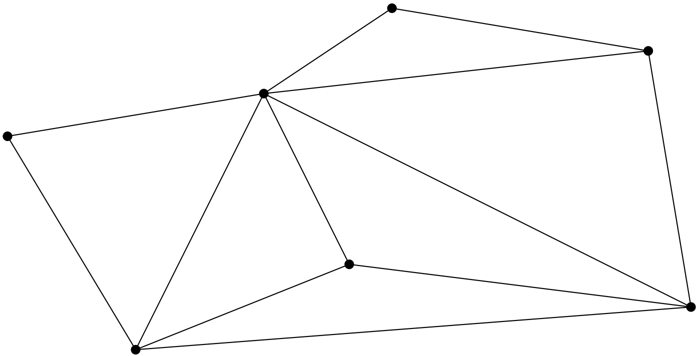

# Geometric Spanners

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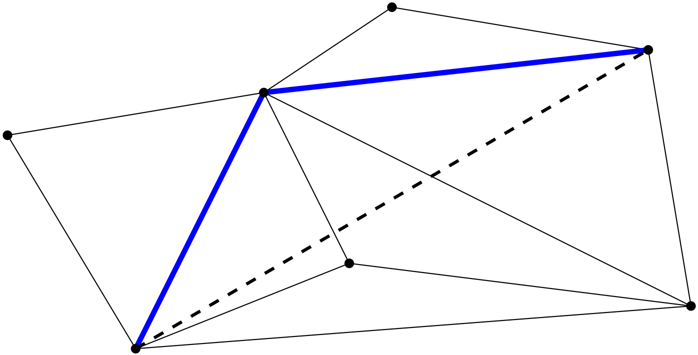
shortest path $\leq k \cdot$ Euclidean distance



# Competitive Routing

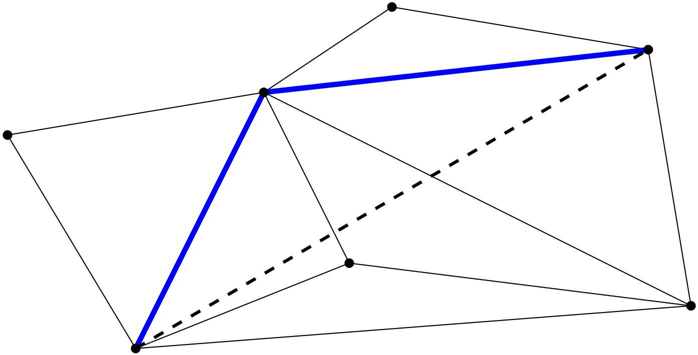
Given:

- Geometric spanner
- Using only local information

Goal:

- Find a short path between any two vertices

path length $\leq r \cdot$ Euclidean distance



# Previous Work

Half-$\theta_6$-graph
(Bonichon et al. 2010)

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Bounded-degree variants
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Bounded-degree variants
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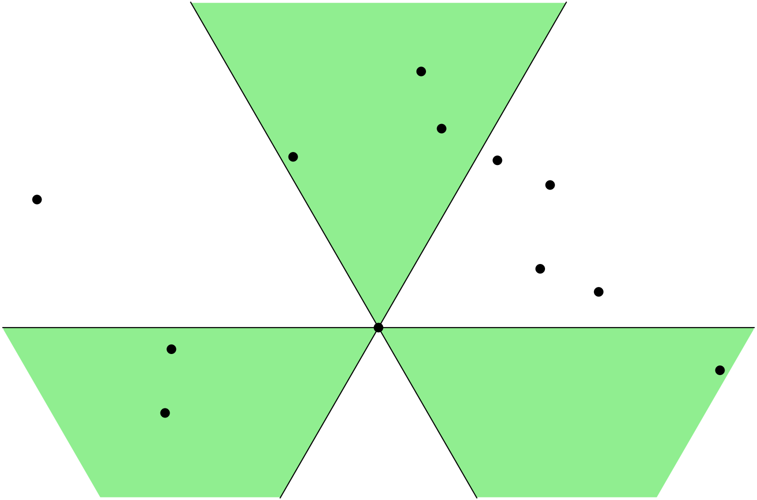
Half-$\theta_6$-graph
(Bonichon et al. 2010)

Competitive routing on
bounded-degree variants
(This result)

Competitive routing
(Bose et al. 2012)

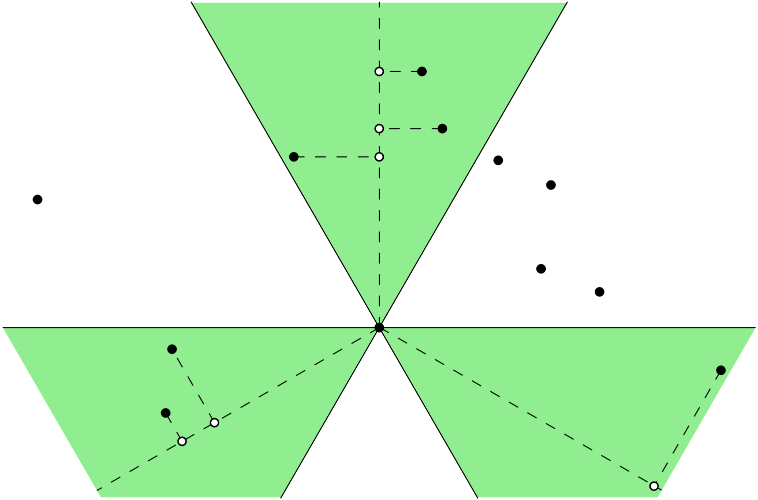
# Half-$\theta_6$-graph

- 6 Cones around each vertex: 3 positive, 3 negative



# Half-$\theta_6$-graph

- Connect to 'closest' vertex in each positive cone



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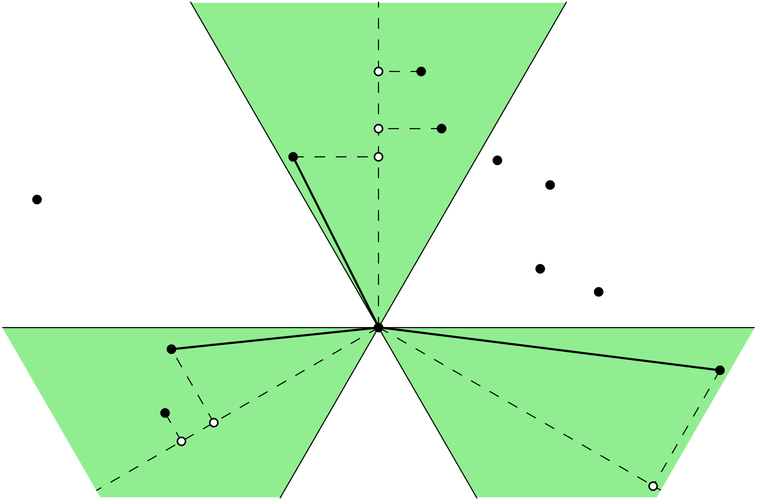

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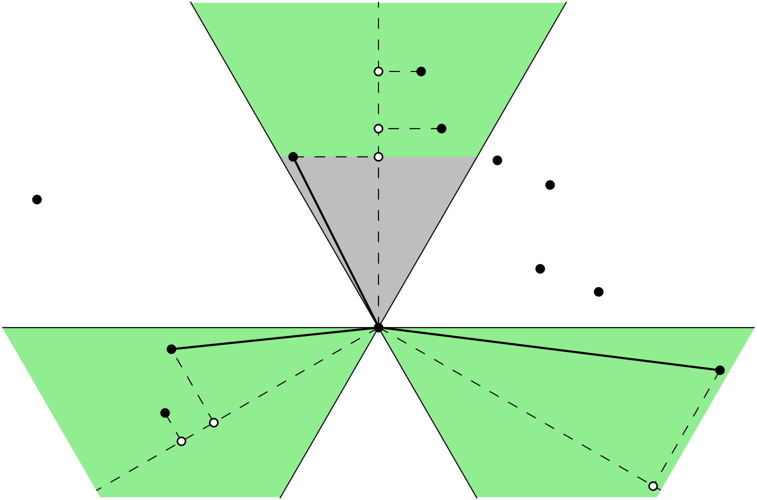
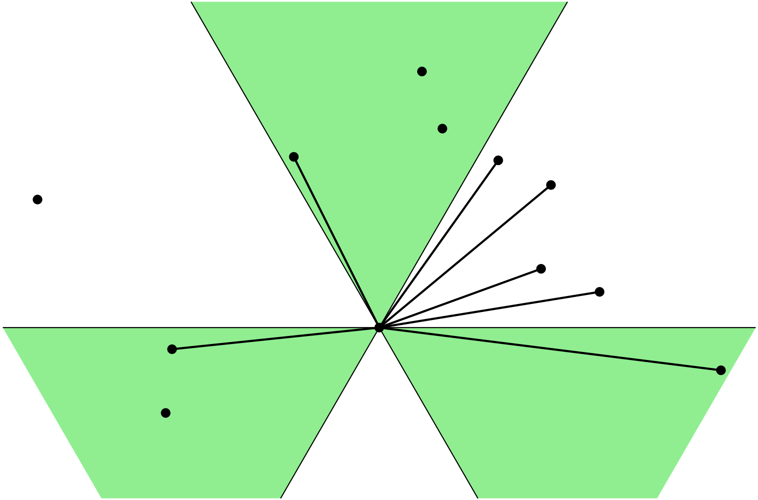


# Bounded Degree

- Negative cones can have unbounded in-degree.



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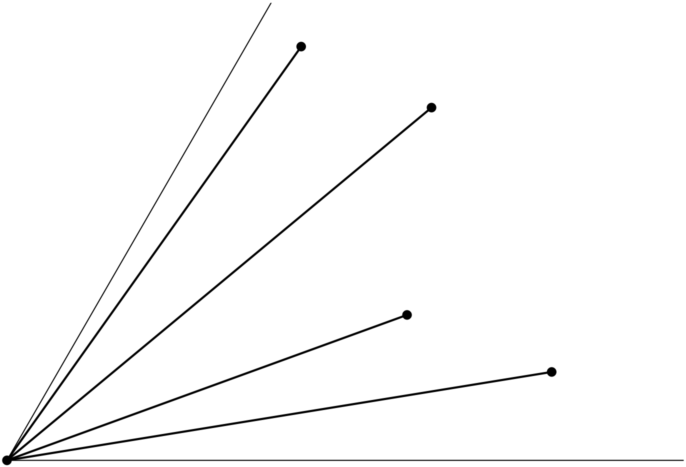
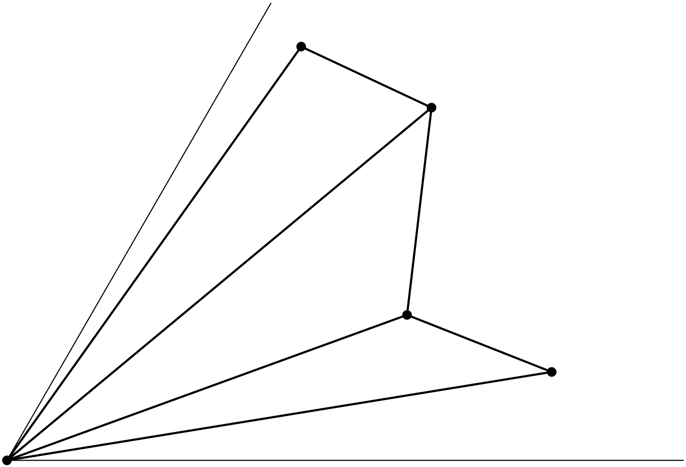


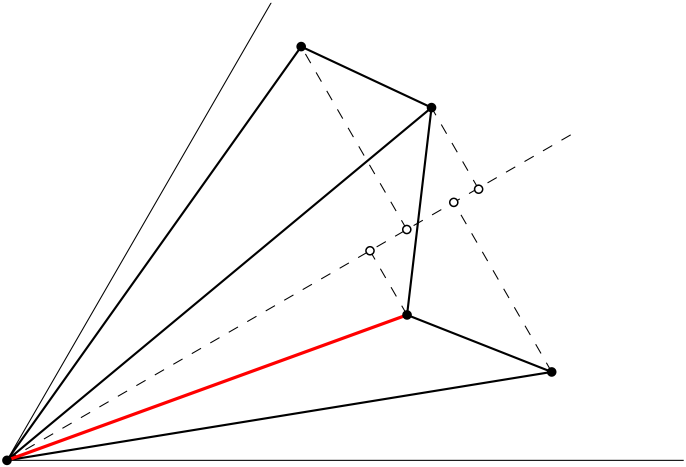
# Bounded Degree

- Consecutive vertices are connected by a *canonical path*.



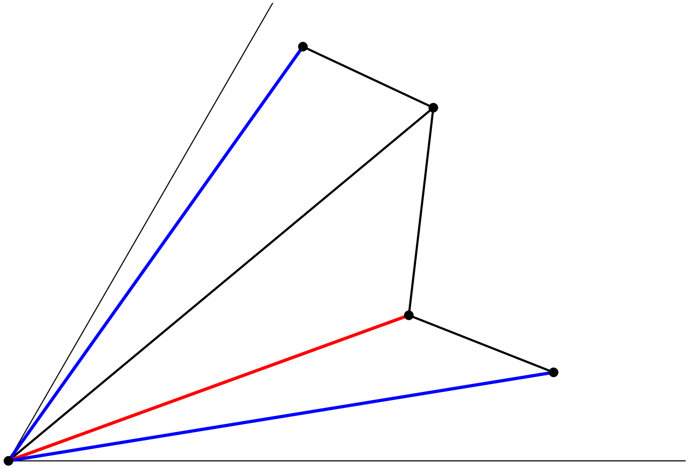
# Bounded Degree

- Keep the edge to the closest vertex...



# Bounded Degree

- Keep the edge to the closest vertex and the extreme edges.



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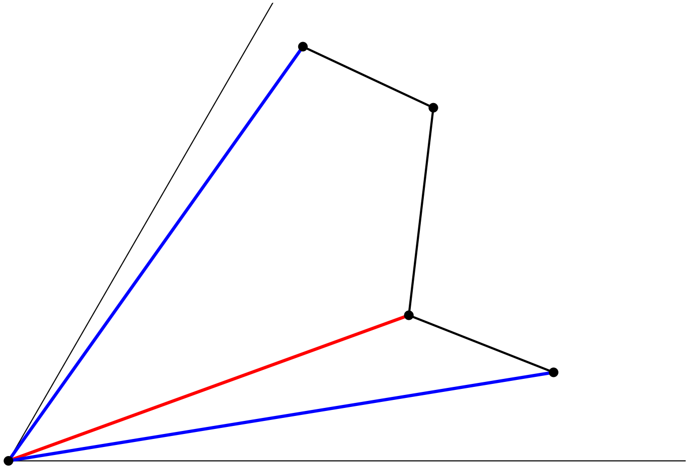
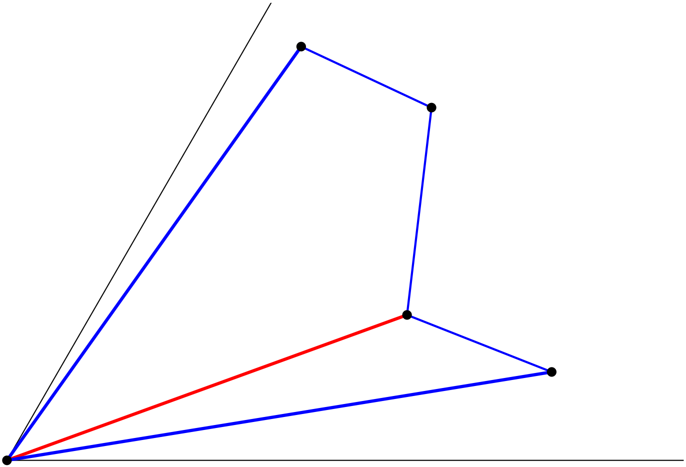


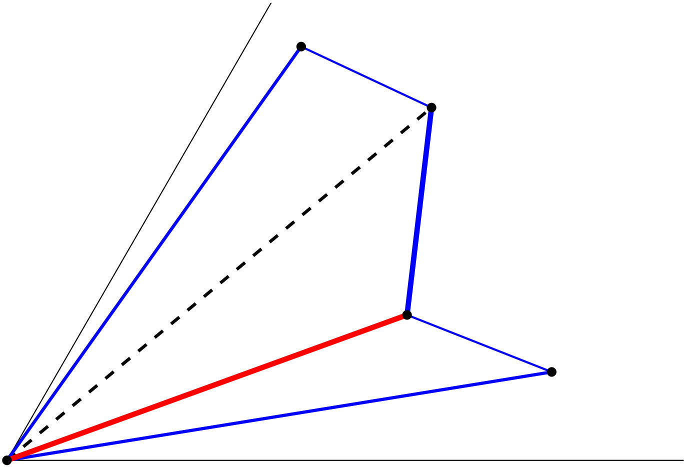
# Bounded Degree

- Edges on the canonical path are always extreme.



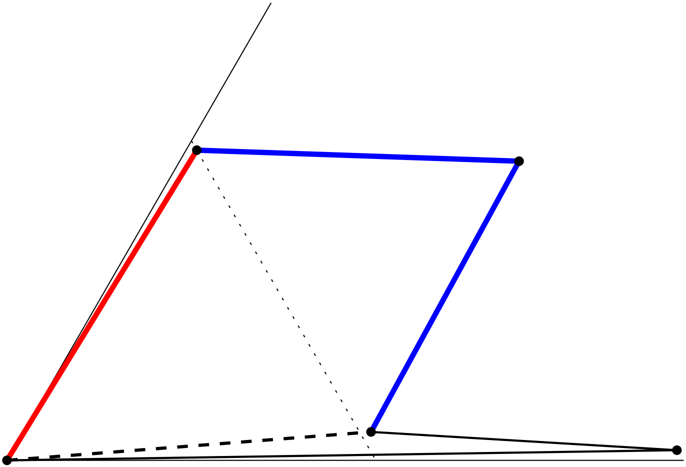
# Bounded Degree

- There is an *approximation path* for every removed edge.



# Bounded Degree

- Result: A 3-spanner of the half-$\theta_6$-graph.

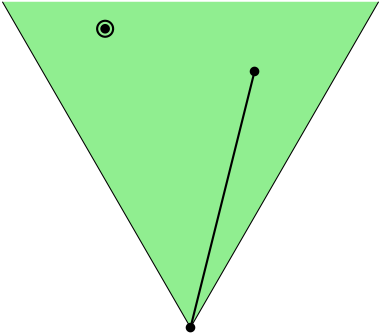


# Routing Algorithm

If $t$ lies in a positive cone:

- Follow the edge in that cone

In the half-$\theta_6$-graph.



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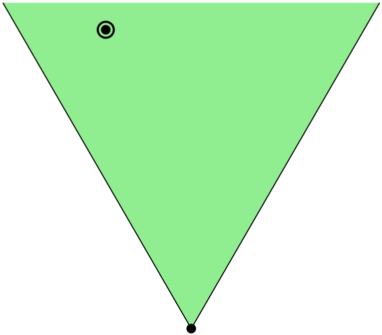

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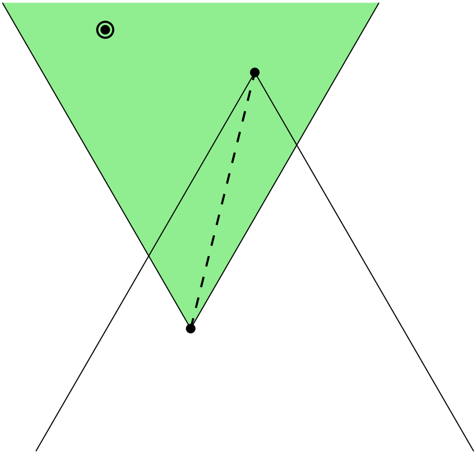

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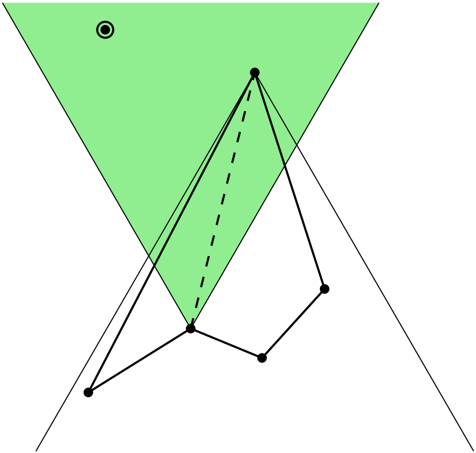

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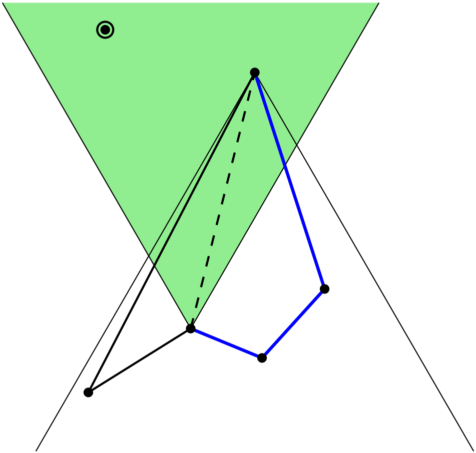

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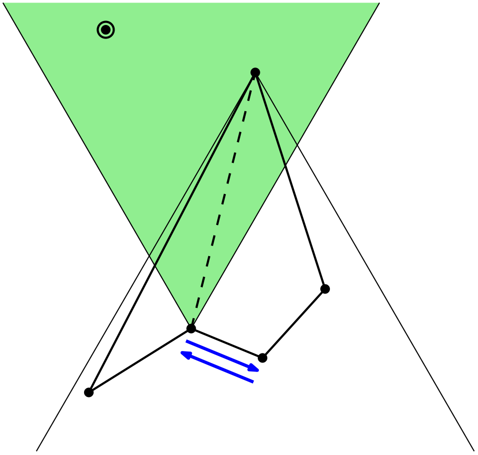

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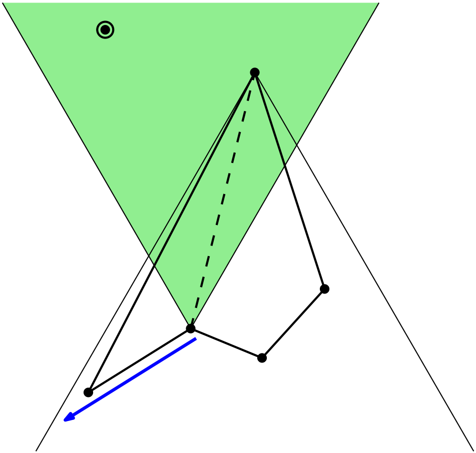

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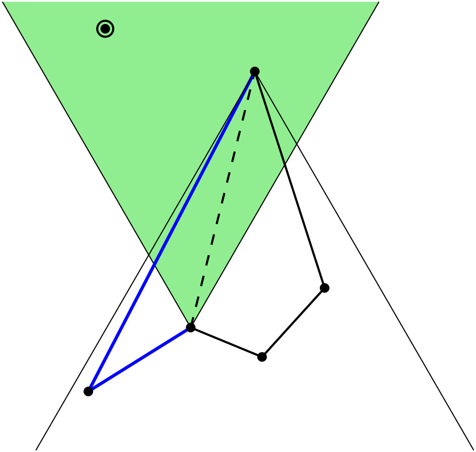

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If $t$ lies in a negative cone and we did not mark a side yet:

- Follow an edge in that cone
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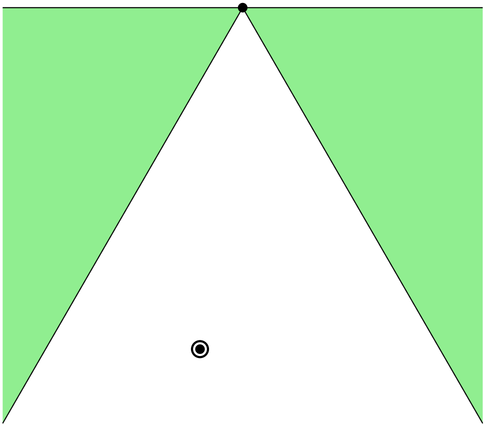
In the half-$\theta_6$-graph.



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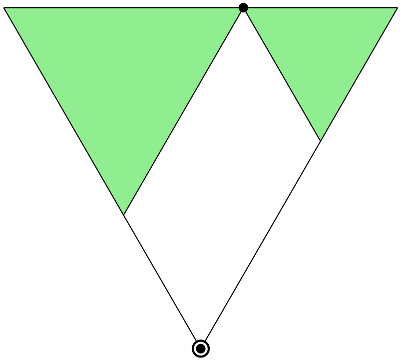

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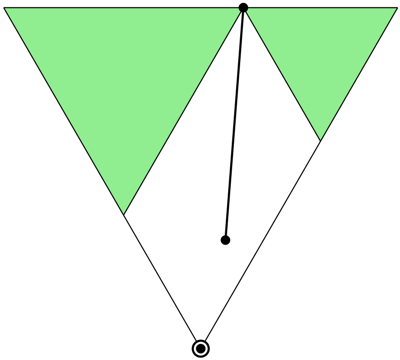

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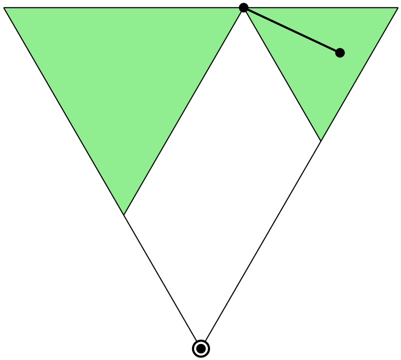

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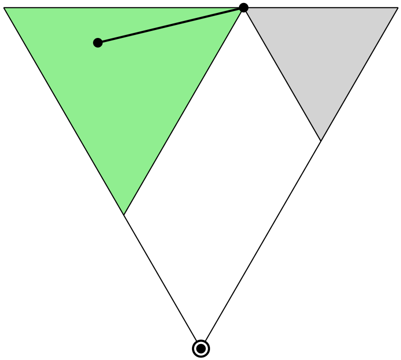

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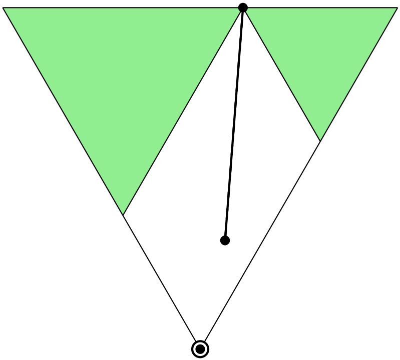

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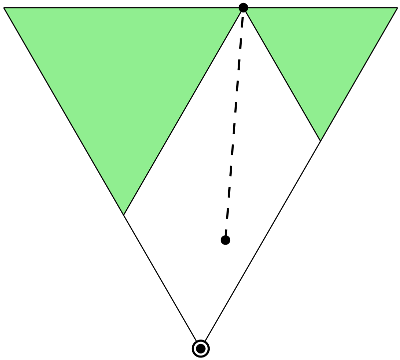

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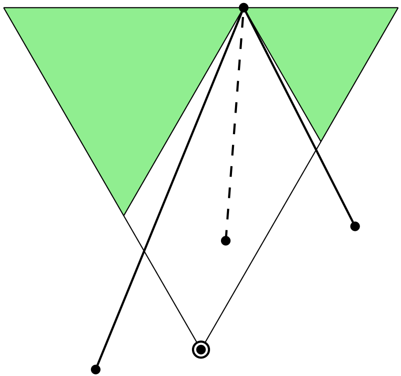

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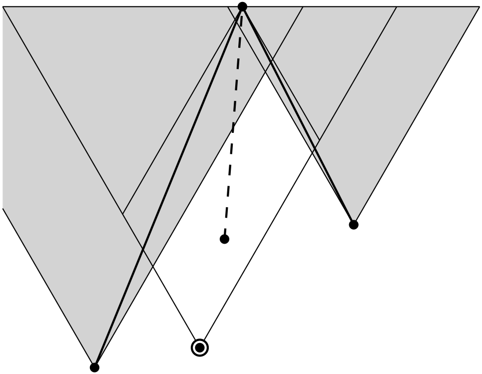

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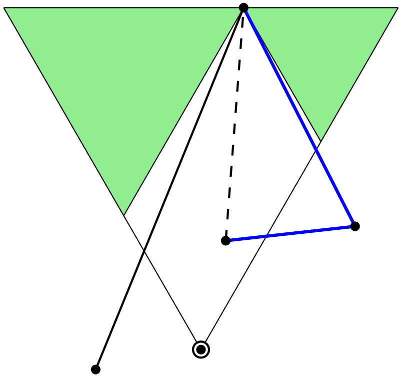

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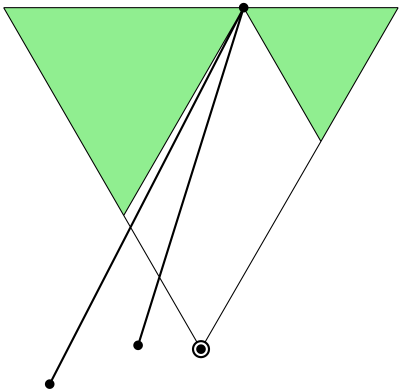

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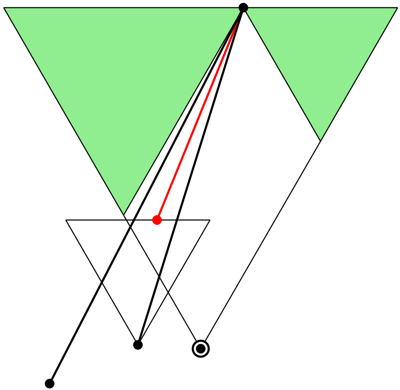

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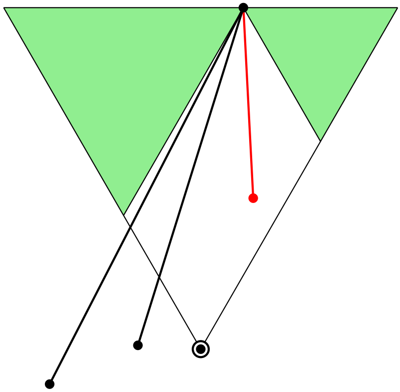

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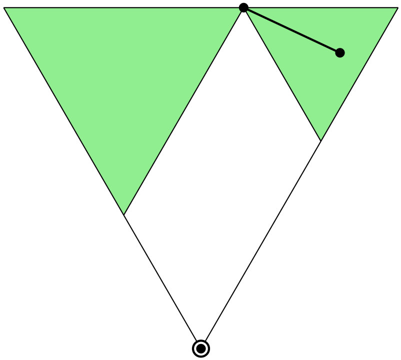

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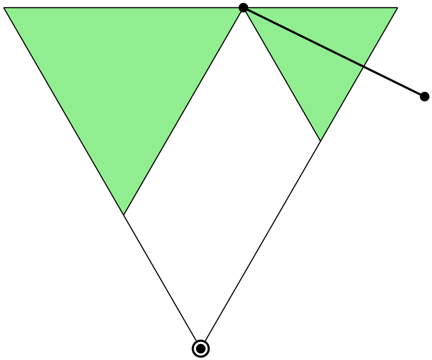

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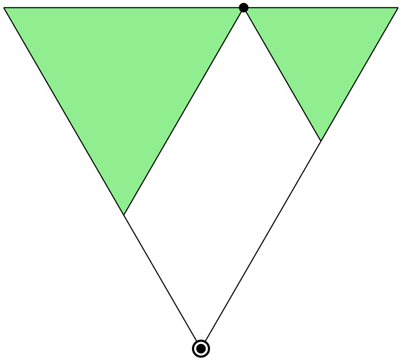

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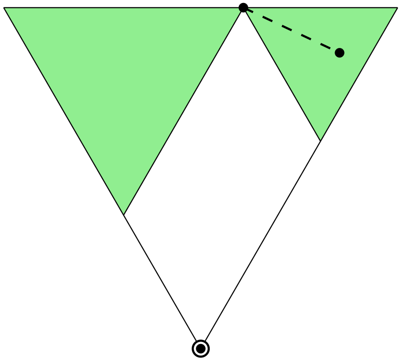

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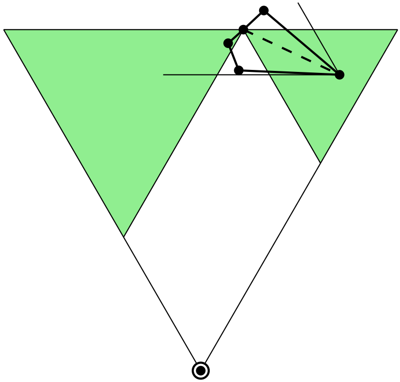

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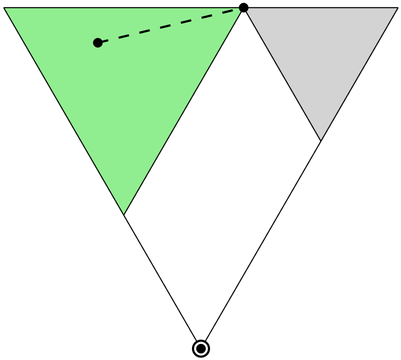

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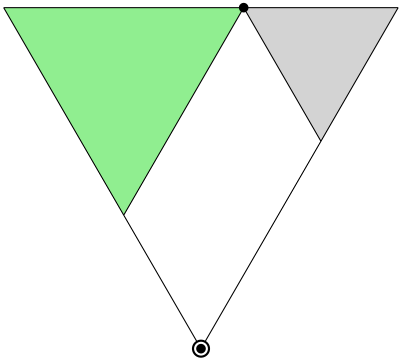
In the half-$\theta_6$-graph.



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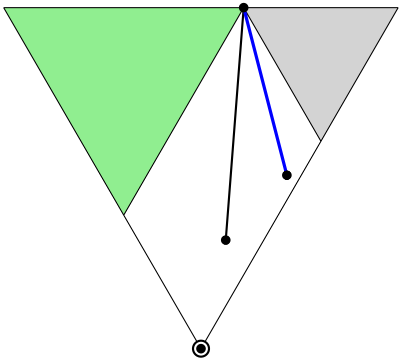

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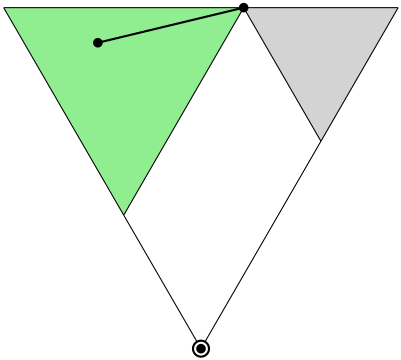

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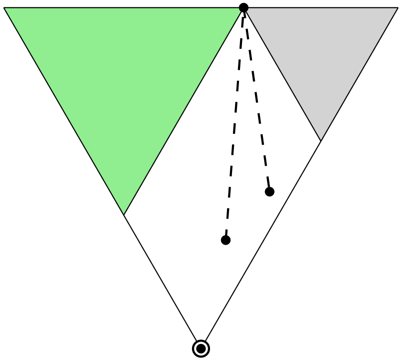

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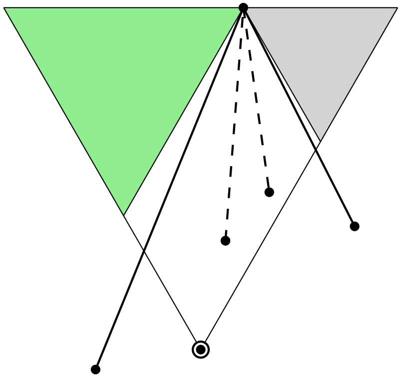

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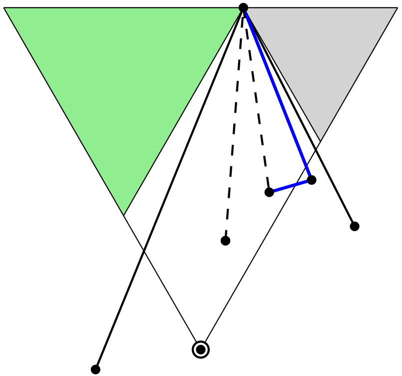

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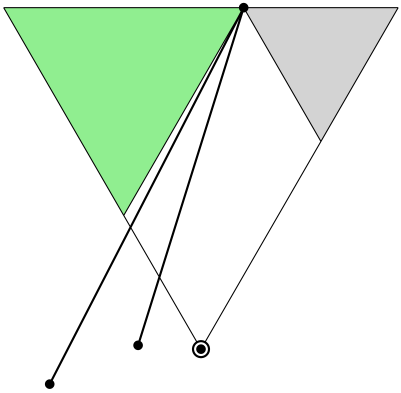

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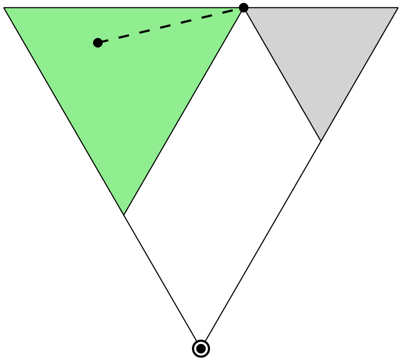

# Conclusion

- Bounded-degree spanners allow competitive routing.

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- Routing ratio can be improved by storing information at vertices.