Power Assignment in Radio Networks with Two Power Levels*

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Abstract

We study the power assignment problem in radio networks, where each radio station can transmit in one of two possible power levels, corresponding to two ranges – short and long. We show that this problem is NP-hard, and present a polynomial-time assignment algorithm, such that the number of transmitters that are assigned long range by the algorithm is at most (11/6) times the number of transmitters that are assigned long range by an optimal algorithm.

1 Introduction

Assigning power levels (corresponding to transmission ranges) to the transmitters of a radio network, so that the total power consumption is as low as possible, is often an extremely important issue. Let $\mathcal{P}$ be a set of n points in the plane, representing n transmitters-receivers (or transmitters for short). We need to assign transmission ranges to the transmitters in $\mathcal{P}$, so that (i) the resulting communication graph is strongly connected; that is, the graph over $\mathcal{P}$ in which there exists a directed edge from p to q if and only if q lies within the transmission range r_p assigned to p, should contain a directed path from any transmitter $p \in \mathcal{P}$ to any other transmitter $q \in \mathcal{P}$, and (ii) the total power consumption (i.e., the cost of the assignment of ranges) is minimized, where the total power consumption is a function of the form $\sum_{p\in\mathcal{P}} r_p^c$, and $c > 0$ is a constant typically between 2 and 5.

This version of the power assignment problem is known to be NP-hard; Kirousis et al. [10] first proved this for 3-dimensional point sets and Clementi et al. [8] then proved this also for planar point sets. Kirousis et al. also present a 2-approximation algorithm, based on the minimum spanning tree of $\mathcal{P}$, which is the best approximation known.

In practice, it is usually impossible to assign arbitrary power levels (ranges) to the transmitters of a radio network. Instead one can only choose from a constant number of preset power levels, corresponding to a constant number of ranges. In this paper we consider the power assignment problem in radio networks, where each transmitter can transmit in one of two given power levels – low or high, corresponding to two possible ranges – short (r_1) and long (r_2). Since the cost of an

*Research by P. Carmi is partially supported by a Kreitman Foundation doctoral fellowship. Research by M. Katz is partially supported by grant No. 2000160 from the U.S.-Israel Binational Science Foundation.

assignment of power levels to the transmitters is a function of the form $n_1 r_1^c + (n - n_1) r_2^c$, where n_1 is the number of transmitters that are assigned range r_1 and $c \geq 1$ is some constant, the cost of an assignment is determined solely by the number of transmitters that are assigned range r_2.

In Sect. 4 we prove that the power assignment problem with two power levels is NP-hard when $r_2 > \sqrt{\frac{3}{2}} r_1$, by constructing a reduction from planar cubic vertex cover. More precisely, we show this for the special case where the initial components graph (see below) is a star.

Let m be the number of transmitters that are assigned range r_2 in an optimal assignment OPT. In Sect. 2 we describe a polynomial-time algorithm that assigns range r_2 to at most $(11/6)m$ transmitters, or in other words, this algorithm computes an (11/6)-approximation (with respect to the number of transmitters that are assigned long range).

An immediate corollary of this result is that for *any* ranges r_1, r_2 and for *any* c, we can compute an assignment whose cost is at most (11/6) times the cost of an optimal assignment. Usually though the cost of our assignment is much less than this, as is shown in Sect. 2.1. In this section we analyze the common case where $r_1 = 1$ and $r_2 = d$. Our algorithm computes in this case an assignment whose cost is at most $\frac{11d^c}{6d^c+5}$ times the cost of an optimal assignment. Plugging for example $d = 2$ we get a $44/29 \approx 1.52$ approximation, if $c = 2$, and a $22/17 \approx 1.29$ approximation, if $c = 1$.

A by-product of our range assignment algorithm is an algorithm for assigning ranges in the special case where the initial components graph is a tree. That is, consider the connected components of the communication graph that is obtained after assigning short range to all transmitters in $\mathcal{P}$. We draw an edge between two components C_1 and C_2 if and only if there exists transmitters $p_1 \in C_1$ and $p_2 \in C_2$, such that the distance between them is at most r_2. Now if this graph happens to be a tree then the algorithm described in Sect. 3 assigns long range to at most $(4/3)m$ transmitters, where m is the number of transmitters assigned long range by an optimal algorithm.

More related work. Other variants of the power assignment problem have been studied. One such variant is the *symmetric* power assignment problem, where the corresponding communication graph is undirected and there exists an edge between two transmitters p and q if and only if both transmitters were assigned ranges greater than (or equal to) the distance between them; see [2, 3, 4]. Clementi et al. [6] consider the problem of assigning ranges to a set of transmitters on a common line, so that for any two transmitters p and q there exists a path from p to q of at most h *hops* in the corresponding (directed) communication graph. The case where $h = n - 1$ was also considered by [10]. An important related problem is the minimum-energy broadcast tree problem: Assign ranges to the transmitters so that a designated source transmitter can broadcast messages to all other transmitters; see, e.g., [5, 7, 11, 12, 14, 15, 16].

2 An (11/6)-Approximation

Let $\mathcal{P}$ be a set of n points in the plane representing n transmitters-receivers (or transmitters for short), and assume that each transmitter can transmit in one of two possible power levels – low or high, corresponding to short range (r_1) or long range (r_2). Further assume that if all transmitters in $\mathcal{P}$ are assigned long range, then the resulting communication graph is strongly connected. In this section we describe a polynomial-time algorithm for assigning ranges to the transmitters in $\mathcal{P}$, such that the number of transmitters that are assigned long range is at most $(11/6)m$, where m is the number of transmitters that are assigned long range by OPT.

Let G be the (undirected) graph of components. G is defined as follows. Assign to each transmitter in $\mathcal{P}$ short range and draw an edge between two transmitters p and q if $|p, q| \leq r_1$, where $|p, q|$ denotes the Euclidean distance between p and q. We think of each of the connected components in this graph as a subset of $\mathcal{P}$. These subsets are the nodes of the graph G; we shall call them components. We draw an edge between two components C_1 and C_2 of G if there exist transmitters $p \in C_1$ and $q \in C_2$, such that $|p, q| \leq r_2$. See Fig. 1.

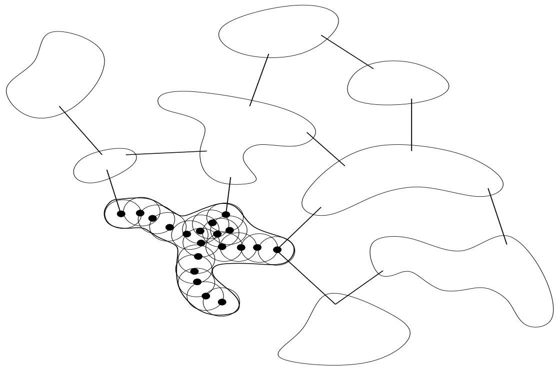

Figure 1: The components graph G.

Notice that we can easily obtain a 2-approximation. Simply compute a minimum spanning tree of G, and, for each edge (C_1, C_2) of the tree, assign long range to two transmitters $p \in C_1$ and $q \in C_2$, such that $|p, q| \leq r_2$.

Our range assignment algorithm consists of two stages. In the first stage we repeatedly find a cycle in G and reduce it to a single component by assigning long range to one transmitter in each of the components in the cycle. The second stage begins when there are no more cycles in G, i.e., when G is a tree. In this stage we assign long range to some more transmitters in order to complete our task.

We now describe the first stage in detail. While there is a cycle in G do the following. Let $C_1, C_2, \ldots, C_l, C_1$ be any cycle of size $l \geq 3$. Assign long range to any transmitter in C_1 that can reach a transmitter in C_2, assign long range to any transmitter in C_2 that can reach a transmitter in C_3, etc. All together we assign long range to l transmitters. Notice that after doing so any two transmitters in the union $C = C_1 \cup \cdots \cup C_l$ can talk with each other possibly through other transmitters in C. Thus these l components reduce to a single component C and the number of components decreases by $l - 1$. We update the graph G by replacing $C_1, \ldots, C_l$ with the single component C. After doing so we forget that some of the transmitters in C have already been assigned long range, and update the edges in G accordingly, see Fig. 2.

At this point there are no cycles left in G, in other words G is a tree. In the next section we present a range assignment algorithm for the case where the components graph is a tree. This algorithm assigns long range to at most $\frac{4}{3}m_{tree}$ of the transmitters, where m_{tree} is the number of transmitters that are assigned long range by an optimal algorithm for this case. Thus in the second stage we apply the algorithm of the next section to G to complete the range assignment task. We now show that the overall number of transmitters that were assigned long range is bounded by $\frac{11}{6}m$.

Theorem 2.1 *The range assignment algorithm (described above) computes an* $(11/6)$*-approximation in polynomial time.*

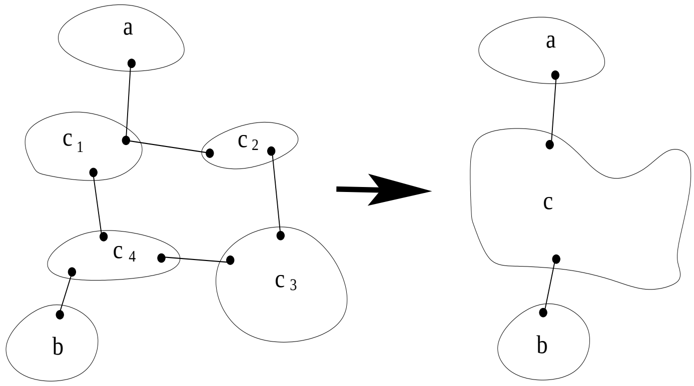

Figure 2: Reducing the cycle C_1, C_2, C_3, C_4, C_1 to the single component C.

Proof: Recall that in the first stage a loop is executed, such that, in each iteration a cycle in G of length at least three is found and replaced by a single component. Let i be the number of cycles that were found during the execution of the loop. We assume that all these cycles are of length exactly three, since this is the worse case for our analysis.

Let k be the initial number of components in G, i.e., right at the beginning of the first stage. Then m, the number of transmitters assigned long range by OPT, is at least k, since in each initial component at least one of the transmitters must be assigned long range. During the first stage the algorithm assigns long range to at most $3i$ transmitters, and the number of components in G at the end of the first stage is $k - 2i$.

At this point G is a tree and we distinguish between two cases.

Case 1: $i > k/2 - m/3$**.** In this case, instead of performing the second stage, we proceed in the most trivial way (for the purpose of the analysis only) and assign long range to $2(k - 2i - 1)$ transmitters. That is, for each edge in G connecting between two components C_1 and C_2, we assign long range to any transmitter in C_1 that can reach a transmitter in C_2 and vise versa. The total number of transmitters that were assigned long range is thus bounded by

$$3i + 2(k - 2i - 1) \leq 2k - i \leq 2k - (\frac{k}{2} - \frac{m}{3}) = \frac{3k}{2} + \frac{m}{3} \leq \frac{11}{6}m \ .$$

Case 2: $i \leq k/2 - m/3$**.** In this case we perform the second stage as described in Sect. 3 and assign long range to at most $(4/3)m_{tree}$ transmitters, where m_{tree} is the number of long range assignments needed to solve the tree G. But clearly $m_{tree} \leq m$, so the number of transmitters assigned long range in the second stage is at most $(4/3)m$. The total number of transmitter that were assigned long range is thus bounded by

$$3i + \frac{4m}{3} \leq 3(\frac{k}{2} - \frac{m}{3}) + \frac{4m}{3} = \frac{3k}{2} + \frac{m}{3} \leq \frac{11}{6}m \ .$$

Since in both cases we were able to bound the total number of long range assignments by $(11/6)m$, we conclude that our range assignment algorithm computes an $(11/6)$-approximation. $\square$

Recall that the cost of an assignment is $n_1 r_1^c + (n - n_1) r_2^c$, where n_1 is the number of transmitters that are assigned range r_1 and $c \geq 1$ is some constant typically between 2 and 5. An immediate corollary of Theorem 2.1 is that for *any* ranges r_1, r_2 and for *any* c, we can compute an assignment whose cost is at most (11/6) times the cost of an optimal assignment. Usually though the cost of our assignment is much less than this, as is shown below.

2.1 The Cost for Ranges 1 and d

Theorem 2.2 *If $r_1 = 1$ and $r_2 = d$, then one can compute a range assignment whose cost is at most $\frac{11d^c}{6d^c+5}$ times the cost of an optimal assignment. For $d = 2$ we get a (44/29)-approximation, if $c = 2$, and a (22/17)-approximation, if $c = 1$.*

Proof: The cost of an optimal algorithm is $d^c \cdot m + 1 \cdot (n - m) = n + (d^c - 1)m$, where m is the number of transmitters assigned long range. We apply both our algorithm and the naive algorithm which assigns range d to all the transmitters. Put $a = n/m$. We distinguish between two cases.
Case 1: $a \leq 11/6$. In this case we use the naive algorithm whose cost is $d^c n$. The ratio between the cost of the naive algorithm and the cost of an optimal algorithm is

$$\frac{d^c n}{n + (d^c - 1)m} \leq \frac{d^c n}{n + (d^c - 1)(6/11)n} = \frac{d^c}{(6/11)d^c + (5/11)} .$$

Case 2: $a \geq 11/6$. In this case we run our algorithm whose cost is

$$d^c \cdot (11/6)m + 1 \cdot (n - (11/6)m) = n + (11/6)(d^c - 1)m .$$

The ratio between the costs is

$$\frac{n + (11/6)(d^c - 1)m}{n + (d^c - 1)m} = \frac{a + (11/6)(d^c - 1)}{a + (d^c - 1)} \leq \frac{(11/6)d^c}{d^c + 5/6} = \frac{d^c}{(6/11)d^c + (5/11)} .$$

In both cases we got a $\frac{d^c}{(6/11)d^c+(5/11)}$-approximation on the cost. Thus for $d = 2$ we get a (44/29)-approximation, if $c = 2$, and a (22/17)-approximation, if $c = 1$. □

3 A (4/3)-Approximation for a Tree of Components

In this subsection we present a (4/3)-approximation algorithm for the case where the components graph G is cycle free, i.e., where G is a tree. In particular G may be the graph that is obtained at the end of the first stage of the general algorithm above.

We first pick an arbitrary component in G to be the root of G. Given a component C in G, we can now refer to its children components and to its parent component in the regular meaning.

For each component C we need to assign long range to some of the transmitters in C, so that for each child C' of C at least one of the transmitters in C assigned long range can reach (a transmitter in) C', and also at least one of these transmitters can reach the parent of C. A neighbor (i.e., one of the children or the parent) C' of C is *satisfied* if at least one of the transmitters in C that can reach it when assigned long range is assigned long range.

Initially all neighbors of C are *unsatisfied.* Our goal is to assign long range to a small number of transmitters in C so that all neighbors of C are satisfied. One can view this problem as a set cover problem: For each transmitter p in C let C_p be the subset of the neighbors of C that can be reached from p by assigning long range to p. It is easy to verify that the size of C_p is at most 5 (since no two components in C_p can be neighbors in G). Thus we could apply known results for k-set cover to achieve our goal; however, this would lead to a weaker result than the one that we obtain below.

We start with the leaf components. The case of a leaf component C is very simple; we assign long range to any transmitter in C that can reach the parent of C (when it is assigned long range). After considering all leaf components, we consider the internal components, where an internal component may be considered only if all its children have already been considered.

Let C be the internal component that is about to be considered. Let χ_C be the number of children of C. Clearly for each child C' of C, we must assign long range to at least one of the transmitters in C' that can reach C (after it is assigned long range). Let m'_C be the number of long range assignments (to transmitters in C) needed to satisfy all children of C. Then m_C, the number of long range assignments (to transmitters in C) assigned by OPT, is either m'_C, if the m'_C transmitters satisfying the children of C can be chosen so that one of them also satisfies the parent of C, or $m_C = m'_C + 1$, otherwise. The following inequalities are immediate: $\sum_C m_C = m$, where m is the overall number of long range assignments assigned by OPT, and $\sum_C \chi_C = k - 1 < m$, where k is the number of components in G. We will assign long range to at most $\frac{1}{3}\chi_C + m_C$ transmitters in C. Summing over all components in G we obtain

$$\sum_C (\frac{1}{3}\chi_C + m_C) \leq \frac{1}{3}m + m = \frac{4}{3}m \ .$$

For each transmitter p in C, let d_p (the *degree* of p) be the number of *unsatisfied children* of C that would be satisfied if p were assigned long range. Notice that d_p refers only to the children of C and not to its parent. After assigning long range to a transmitter q in C we update the degrees d_p of all transmitters p in C (in particular d_q becomes 0).

We are now ready to describe our algorithm for assigning long range to transmitters in C. If $\chi_C \leq 2$, then we "solve" C optimally, that is, we find a minimum subset of transmitters in C that can reach all children of C and can also reach its parent (when assigned long range). We can do this since in this case $m_C \leq 3$.

Otherwise, as long as the number of unsatisfied children is at least 3 and there exists a transmitter of degree at least 3, we assign long range to any such transmitter q and update the degrees of all transmitters in C accordingly. By assigning long range to q we satisfy at least 3 of the children of C. Since for each of these 3 children, OPT assigns long range to one of their transmitters so that it can reach C, we charge the assignment to q to these 3 assignments of OPT. Thus in this loop we have used at most $\frac{1}{3}(\chi_C - x)$ long range assignments to transmitters in C, where $x \geq 0$ is the number of remaining unsatisfied children of C.

At this point either $x \leq 2$, or $x \geq 3$ and all transmitters in C have degree at most 2. In the former case we "solve" the remaining subproblem optimally (assigning long range to at most $3 \leq m_C$ transmitters in C). We have used in total at most $m_C + \frac{1}{3}\chi_C$ long range assignments.

In the latter case, where we are left with at least 3 unsatisfied children and transmitters of degree at most 2, we first assign long range to any transmitter in C that can reach C's parent (when assigned long range), and update the degrees of the transmitters in C. We charge this assignment to the at least 3 remaining unsatisfied children of C. Next we "solve" the remaining subproblem optimally using the optimal solution to 2-set cover [9]. Again we have used in total at most $m_C + \frac{1}{3}\chi_C$ long range assignments.

Theorem 3.1 *If the components graph G is a tree, one can compute a range assignment that is a (4/3)-approximation in polynomial time.*

4 NP-Hardness

Let r_1 and r_2 be any two ranges, such that $r_2 > \sqrt{\frac{3}{2}} r_1$. In this section we show that the problem of finding an optimal range assignment for a given set $\mathcal{P}$ of points in the plane (representing transmitters-receivers) is NP-Hard. One can think of the problem as follows: Assign short range (r_1) to each of the transmitters in $\mathcal{P}$. The goal now is to find a smallest subset $\mathcal{P}' \subseteq \mathcal{P}$ of transmitters, such that, after assigning long range (r_2) to each of the transmitters in $\mathcal{P}'$, one obtains a strongly connected graph.

Consider the components graph G that is obtained when each transmitter in $\mathcal{P}$ is assigned short range (see Sect. 2 for a precise definition of G). We show that even the special case where G is a star, i.e., G consists of one *central* component C that is connected to k *orbit* components (see Fig. 3) is NP-hard. In this case, the problem is to find a smallest subset of transmitters in C that satisfies all orbit components (when each of the transmitters in the subset is assigned long range).

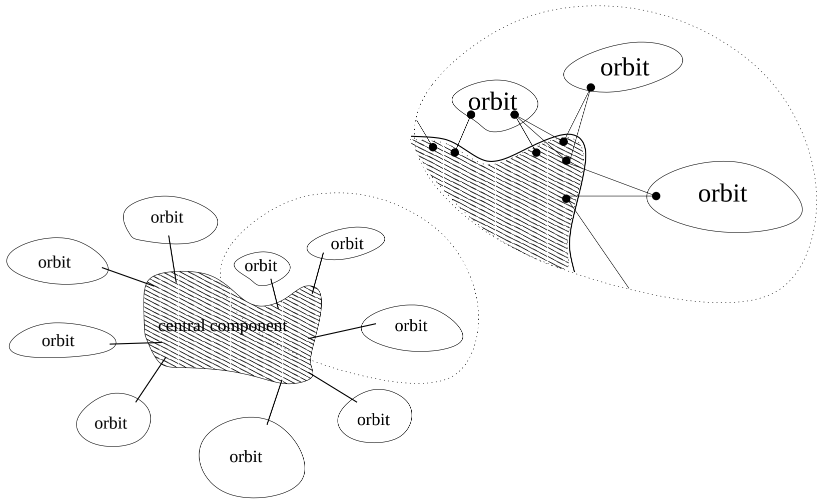

Figure 3: A star graph of components.

We describe a reduction from minimum vertex cover in planar cubic graphs. Let $PCG = (V, E)$ be a planar cubic graph (i.e., each of the nodes in PCG has degree at most 3). A *vertex cover* for PCG is a subset U of V, such that, for each edge $(v_1, v_2) \in E$, either $v_1 \in U$ or $v_2 \in U$. The problem of finding a vertex cover of minimum size in planar cubic graphs is known to be NP-Hard [1, 9].

Valiant [13] showed that any planar cubic graph $PCG = (V, E)$ can be embedded in a rectangular grid of size $O(|V|^2)$ as follows. Each node $v \in V$ corresponds to some grid vertex, and each edge $(v_1, v_2) \in E$ corresponds to a rectilinear path formed of grid edges, whose endpoints are the grid vertices corresponding to v_1 and v_2. Moreover the interiors of any two such paths are disjoint.

We now convert the embedded graph $PCG' = (V', E')$ into a star components graph G, see Fig. 4. We assume that the distance between adjacent grid vertices is $3r_2$. Each edge $e' \in E'$ is

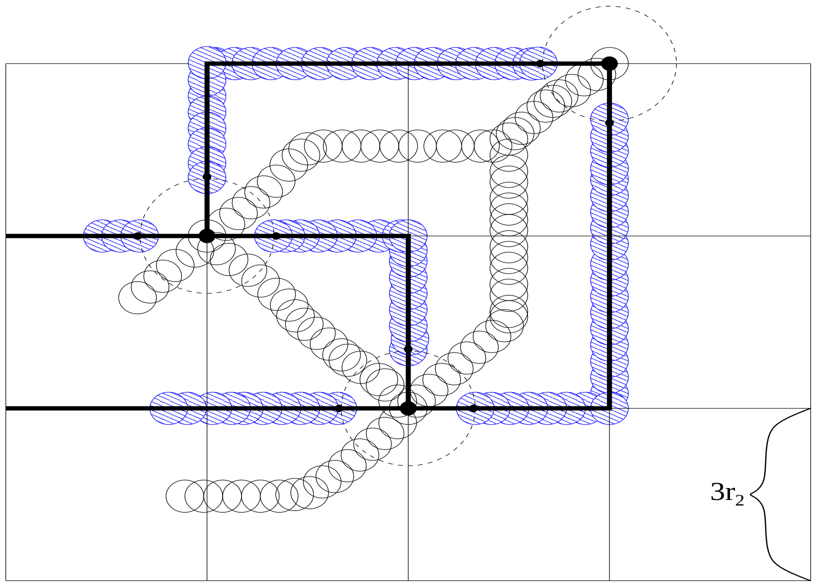

Figure 4: Converting the embedded graph PCG' to a star graph of components.

converted into an orbit component of G, and the set V' is converted into the central component of G. We convert $e' = (v'_1, v'_2) \in E'$ into an orbit component by placing transmitters on the path e' as follows. Place transmitters along the path e' beginning at the point on e' at distance r_2 from v'_1 and ending at the point on e' at distance r_2 from v'_2, such that the distance between any two consecutive transmitters is at most r_1.

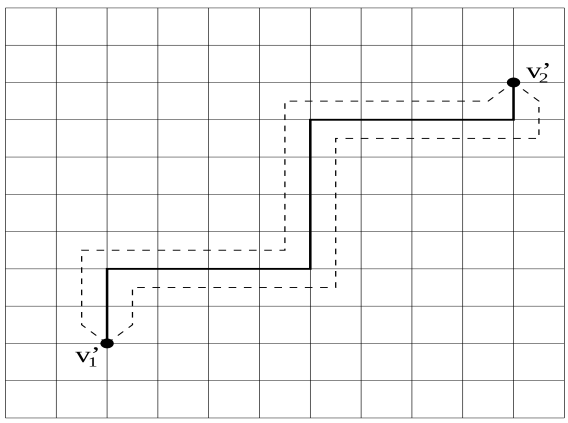

Figure 5: Connecting between v'_1 and v'_2.

We convert the set V' into the central component by placing transmitters as follows. For each edge $e' = (v'_1, v'_2) \in E'$ we place transmitters at v'_1 and at v'_2 and along one of the two dashed paths between them (see Fig. 5), so that the distance between any two consecutive transmitters is at most r_1. The requirement $r_2 > \sqrt{\frac{3}{2}} r_1$ ensures that if we are careful then none of the transmitters along the portion of the dashed path connecting v'_1 (alternatively, v'_2) to the center of the appropriate adjacent grid cell is within distance r_1 of a transmitter belonging to an orbit component. (Notice that we may assume that PCG is connected, since otherwise we could find a minimum vertex cover for each of its connected components and their union would be a minimum vertex cover for PCG.)

It is easy to verify that we obtained a star components graph G. That is (i) a transmitter in an orbit component C' that is assigned long range can either not reach any other component, or can only reach the central component (as is the case for the extreme transmitters in C'), and (ii) for each orbit component C' obtained from the edge $e' = (v'_1, v'_2)$ there exists a transmitter in the central component that can reach C', when assigned long range. The transmitters at v'_1 and at v'_2 are such transmitters.

Moreover, for any transmitter p in the central component, there exists a vertex v' that dominates it, in the sense that if both p and v' are assigned long range, then any orbit component that can be reached from p can also be reached from v'. Therefore when solving the range assignment problem, we may restrict ourselves to vertices v' in the central component. Also the total number of transmitters that were used in the construction is polynomial in n. Finally, an optimal solution for the range assignment problem corresponds to a minimum vertex cover for the graph PCG.

Theorem 4.1 *Let r_1 and r_2 be any two ranges, such that $r_2 > \sqrt{\frac{3}{2}} r_1$. Then the problem of finding an optimal range assignment (where r_1 and r_2 are the two possible ranges) for a given set $\mathcal{P}$ of points in the plane is NP-Hard.*

References

[1] P. Alimonti and V. Kann. Hardness of Approximating Problems on Cubic Graphs. *Proc. 3rd Italian Conf. Algorithms and Complexity*, LNCS 1203, 288–298, 1997.

[2] E. Althaus, G. Călinescu, I. I. Măndoiu, S. Prasad, N. Tchervenski, and A. Zelikovsky. Power Efficient Range Assignment in Ad-hoc Wireless Networks. *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, 1889–1894, 2003.

[3] D. M. Blough, M. Leoncini, G. Resta, and P. Santi. On the Symmetric Range Assignment Problem in Wireless Ad Hoc Networks. *Proc. 2nd IFIP Int. Conf. Theoretical Computer Science*, 71–82, 2002.

[4] G. Călinescu, I. I. Măndoiu, and A. Z. Zelikovsky. Symmetric Connectivity with Minimum Power Consumption in Radio Networks. *Proc. 2nd IFIP Int. Conf. Theoretical Computer Science*, 119–130, 2002.

[5] A. E. F. Clementi, P. Crescenzi, P. Penna, and R. Silvestri. On the Complexity of Computing Minimum Energy Consumption Broadcast Subgraphs. *Sympos. Theoretical Aspects of Computer Science (STACS)*, 121–131, 2001.

[6] A. E. F. Clementi, A. Ferreira, P. Penna, S. Perennes, and R. Silvestri. The Minimum Range Assignment Problem on Linear Radio Networks. *European Sympos. on Algorithms (ESA)*, 143–154, 2000.

[7] A. E. F. Clementi, G. Huiban, P. Penna, G. Rossi, and Y. C. Verhoeven. On the Approximation Ratio of the MST-Based Heuristic for the Energy-Efficient Broadcast Problem in Static Ad-Hoc Radio Networks. *Int. Parallel and Distributed Processing Sympos. (IPDPS)*, 2003.

[8] A. E. F. Clementi, P. Penna, and R. Silvestri. On the Power Assignment Problem in Radio Networks. *Electronic Colloquium on Computational Complexity*, 2000.

[9] M. R. Garey and D. S. Johnson. Computers and Intractability: A Guide to the Theory of NP-Completeness. *W. H. Freeman and Company*, 1979.

[10] L. M. Kirousis, E. Kranakis, D. Krizanc, and A. Pelc. Power Consumption in Packet Radio Networks. *14th Annual Sympos. Theoretical Aspects of Computer Science (STACS)*, LNCS 1200, 363–374, 1997.

[11] W. Liang. Constructing Minimum-Energy Broadcast Trees in Wireless Ad Hoc Networks. *Proc. 3rd ACM International Symposium on Mobile Ad Hoc Networking and Computing*, 112–122, 2002.

[12] I. Papadimitriou and L. Georgiadis. Energy-aware Broadcasting in Wireless Networks. *Proc. Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt)*, 267–277, 2003.

[13] L. Valiant. Universality Considerations in VLSI Circuits. *IEEE Trans. computers 30*, 135–140, 1981.

[14] P.-J. Wan, G. Călinescu, X.-Y. Li, and O. Frieder. Minimum-Energy Broadcast Routing in Static Ad Hoc Wireless Networks. *ACM Wireless Networks* 8:607–617, 2002.

[15] J. E. Wieselthier, G. D. Nguyen, and A. Ephremides. On the Construction of Energy-Efficient Broadcast and Multicast Trees in Wireless Networks. *IEEE INFOCOM*, 585–594, 2000.

[16] J. E. Wieselthier, G. D. Nguyen, and A. Ephremides. Algorithms for Energy-Efficient Multicasting in Static Ad Hoc Wireless Networks. *ACM Mobile Networks and Applications (MONET)* 6(3):251–263, 2001.