Maarten Löffler Marc van Kreveld

IMPRECISE LINES

Center for Geometry, Imaging
and Virtual Environments

Utrecht University

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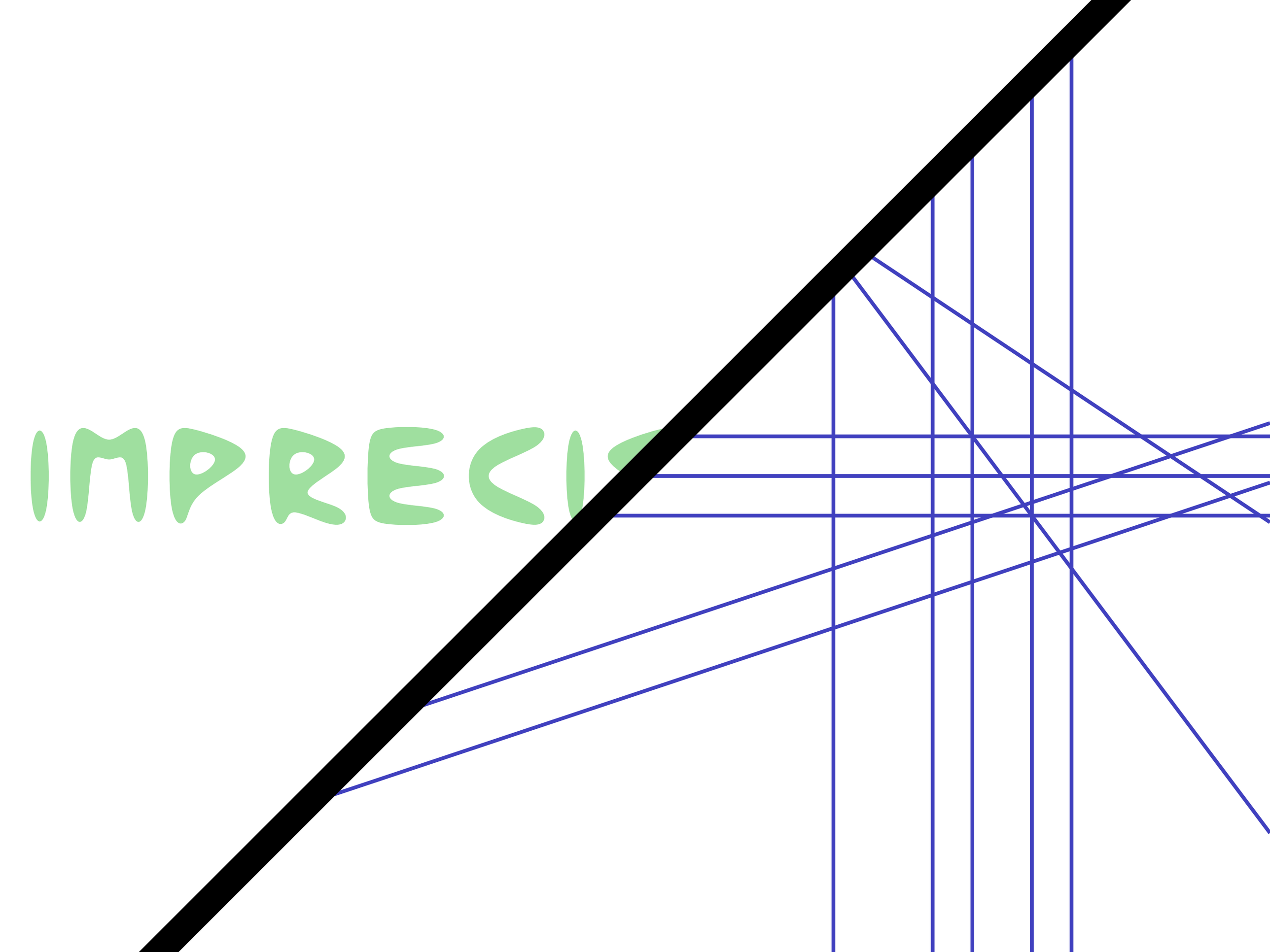

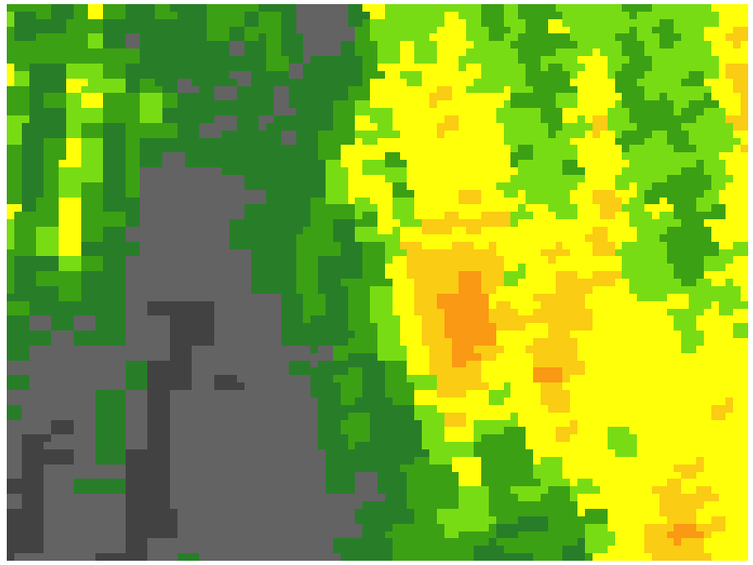

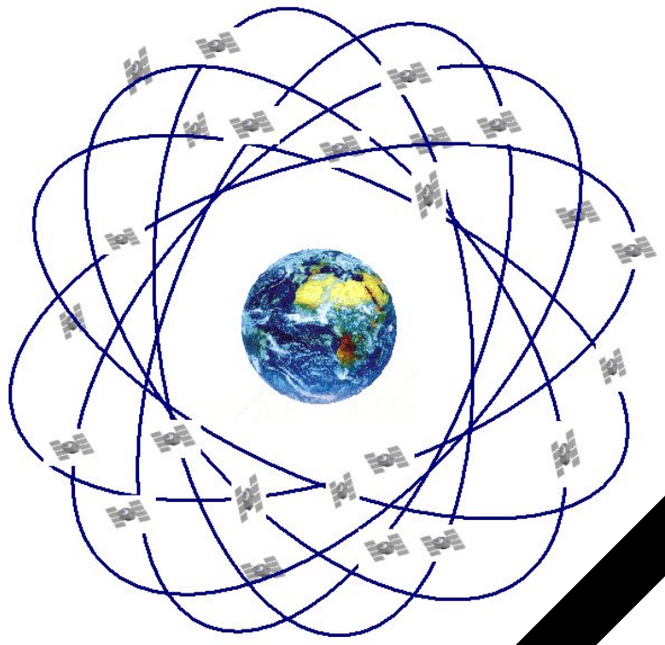

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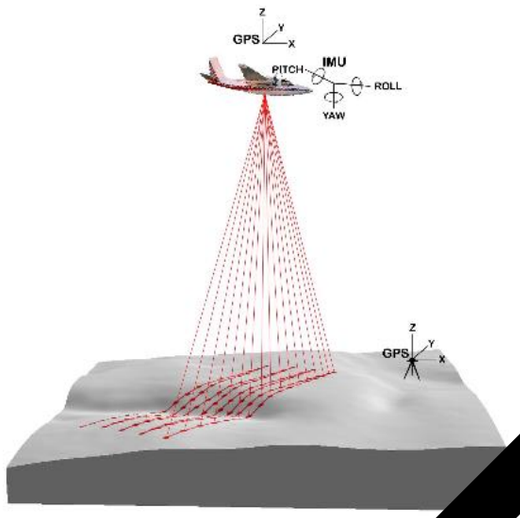

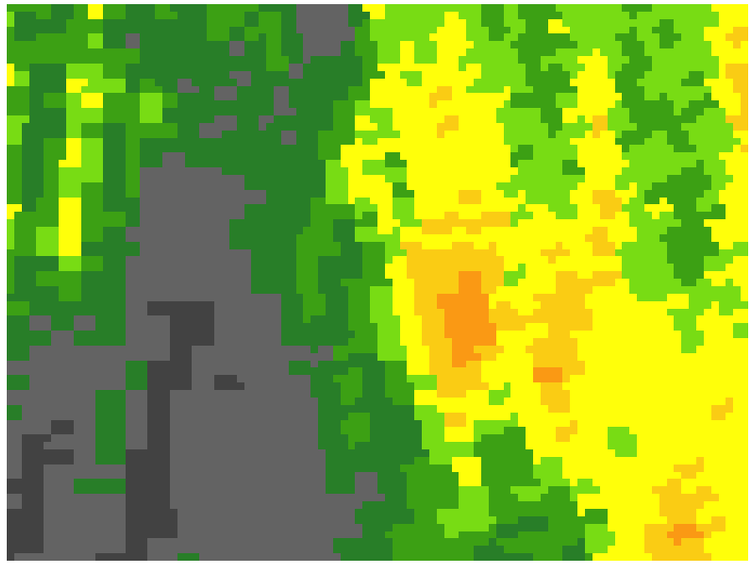

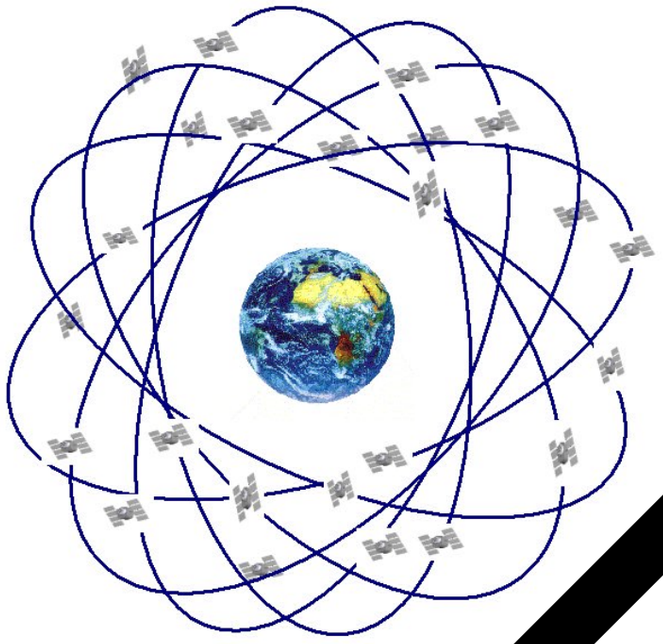

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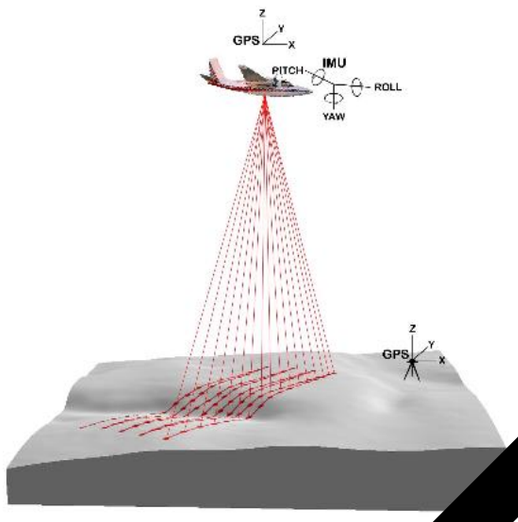

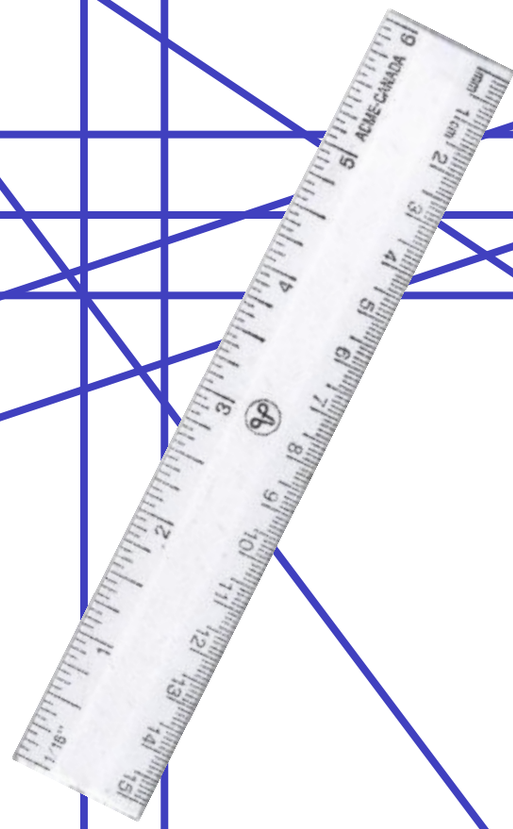

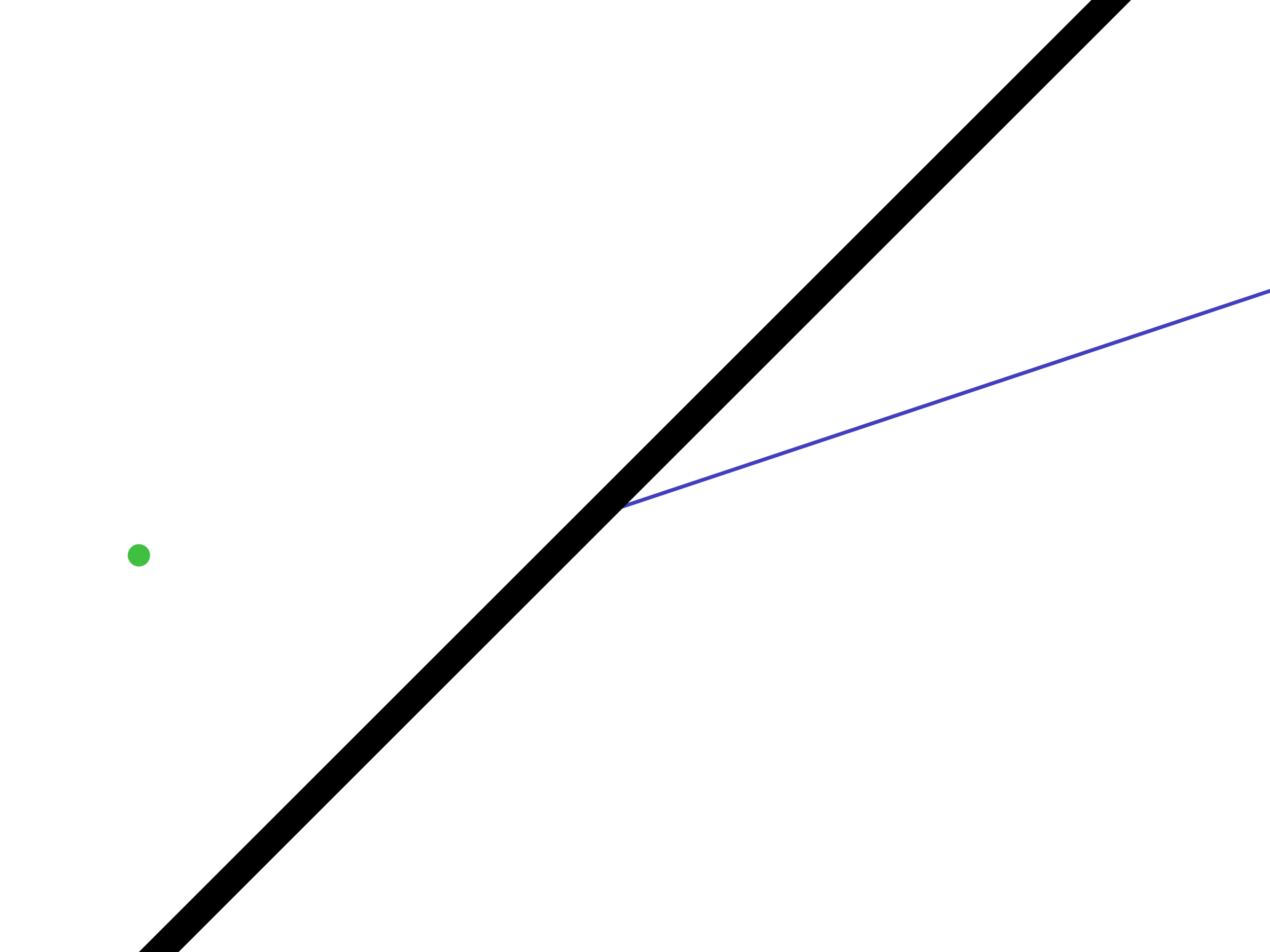

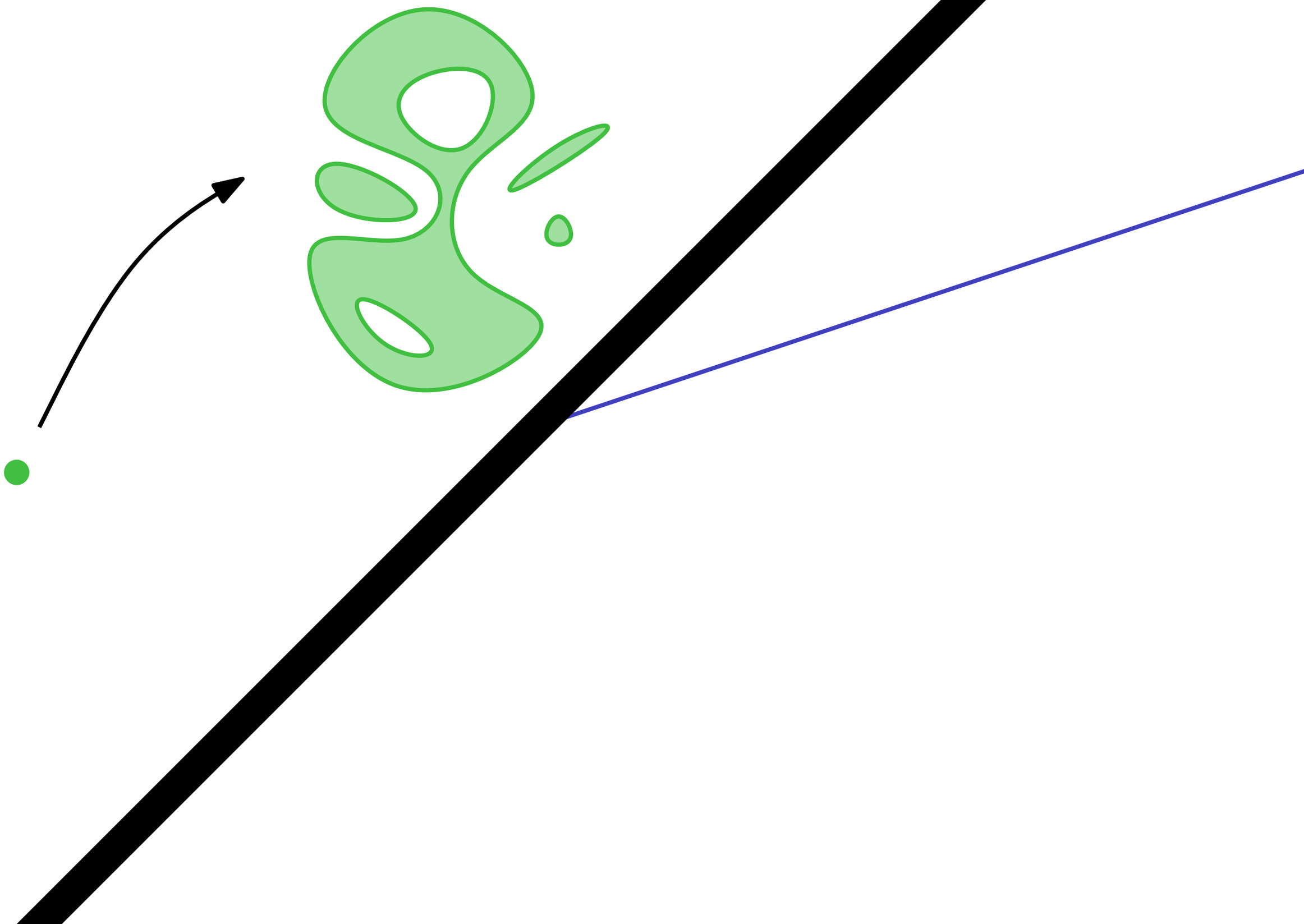

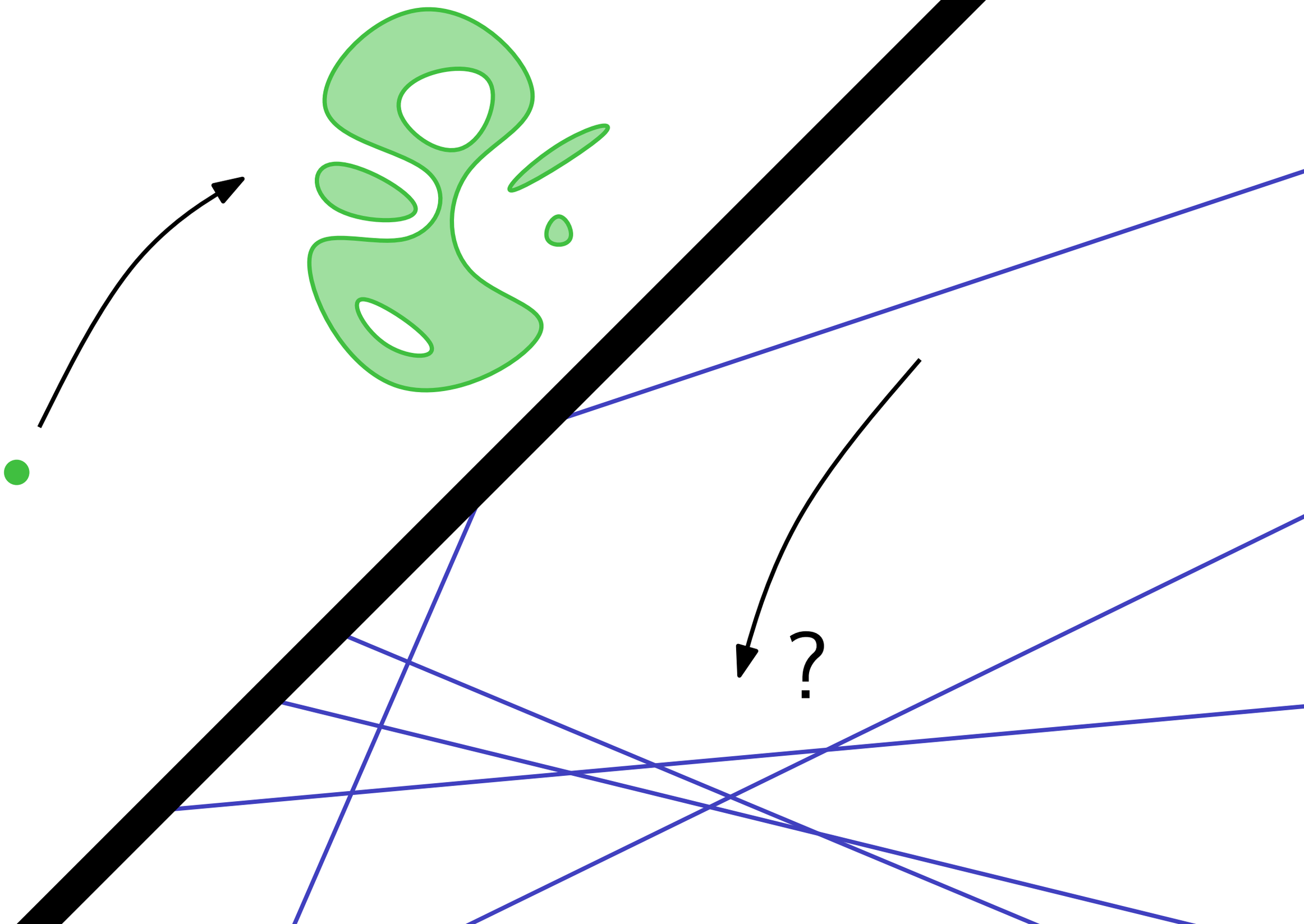

PROPERTIES OF IMPRECISE POINTS

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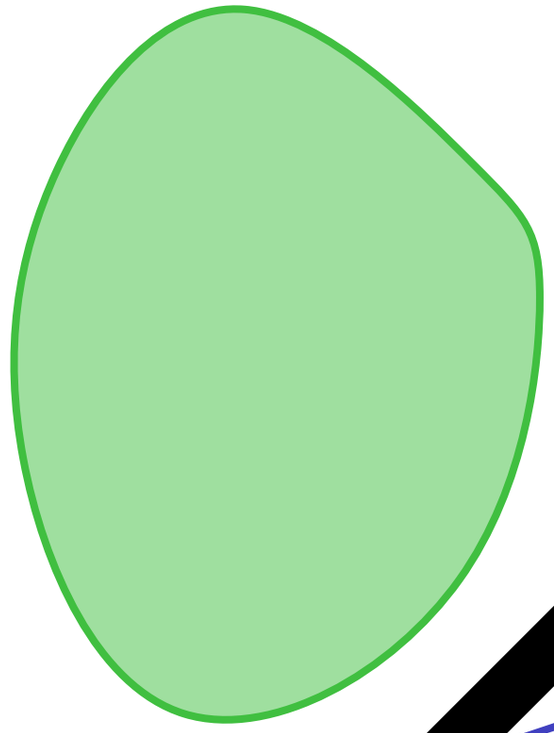

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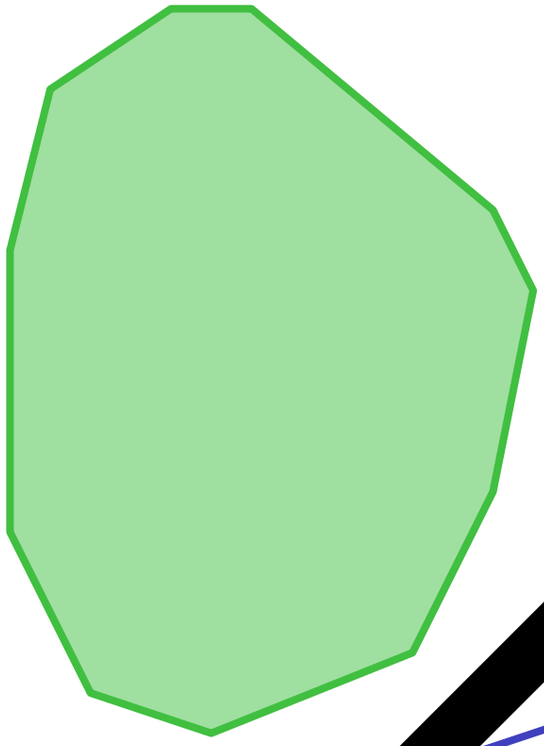

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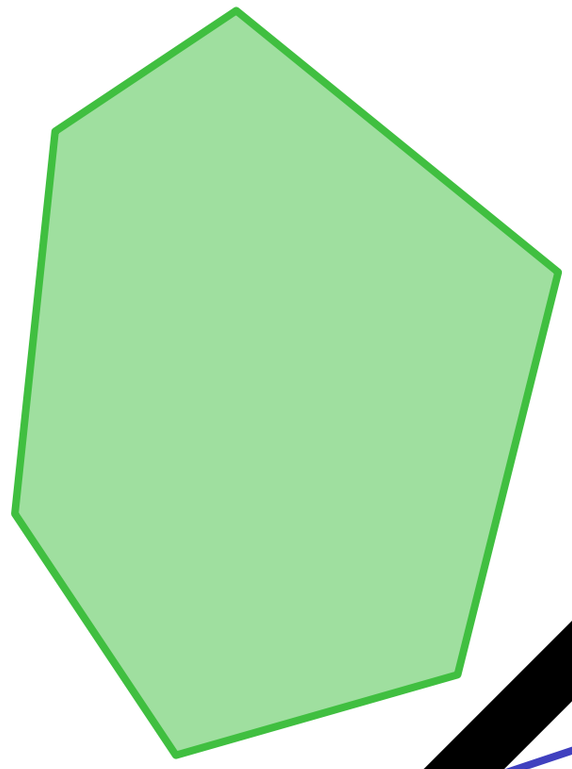

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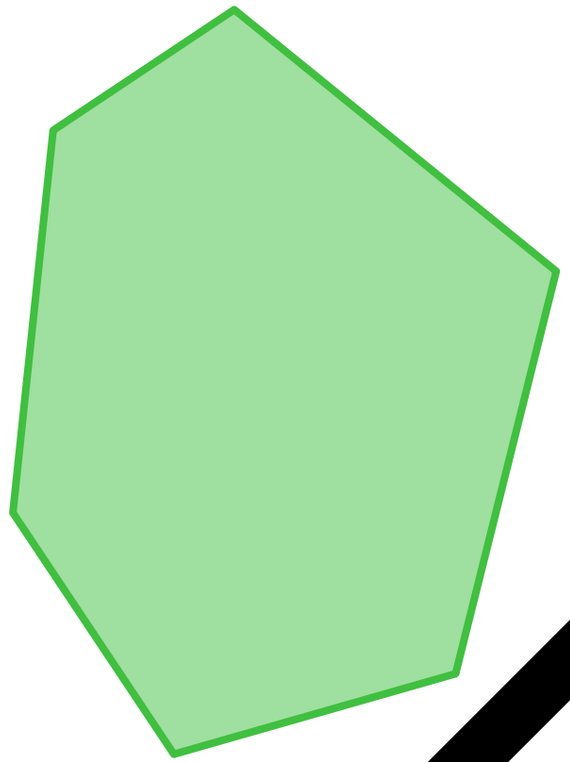

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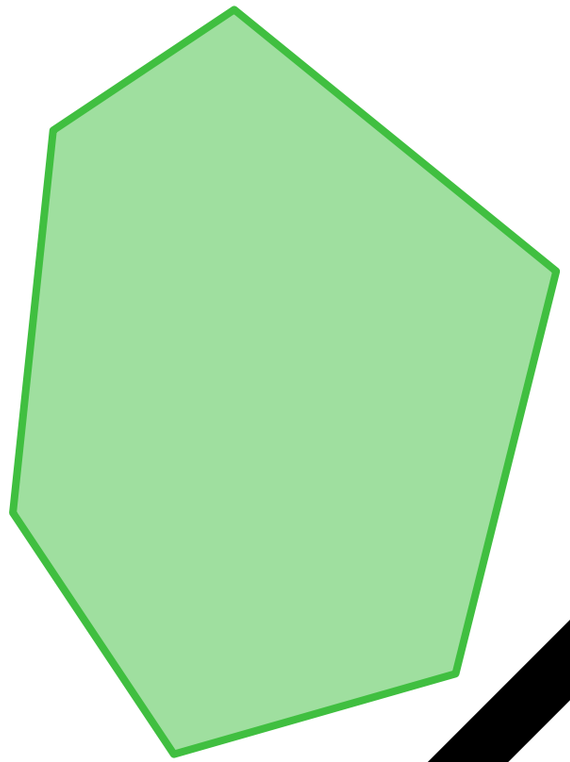

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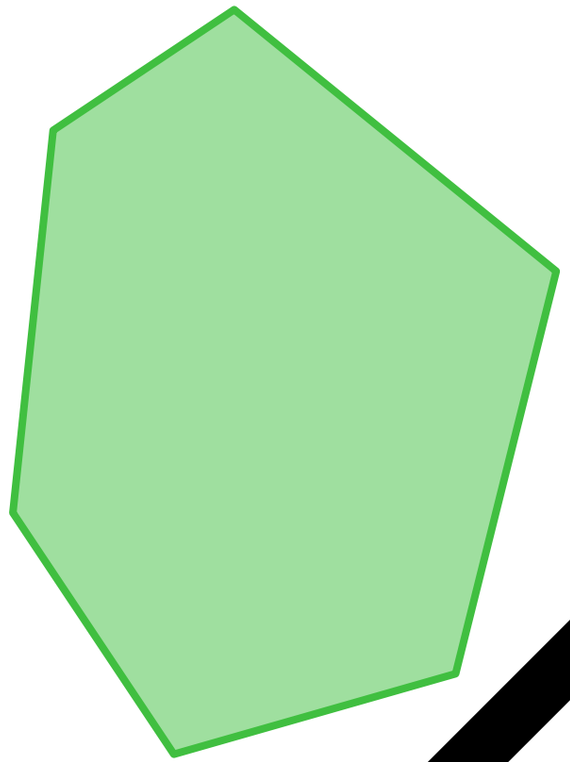

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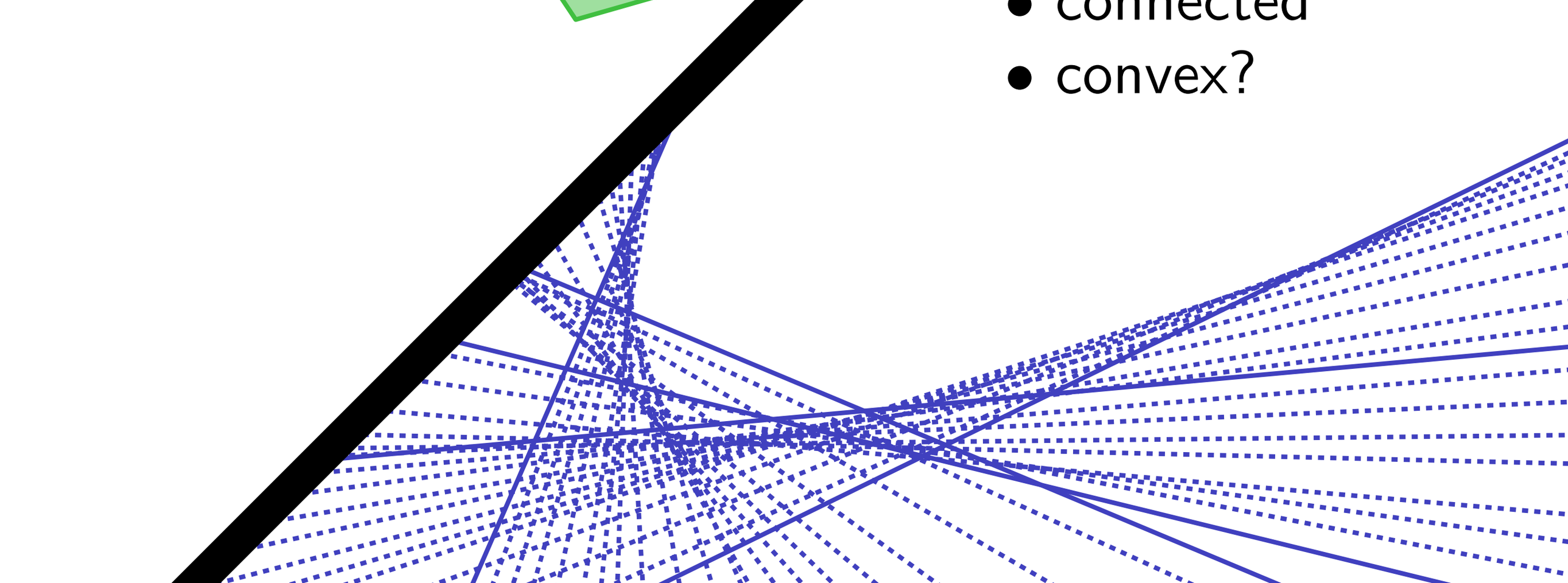

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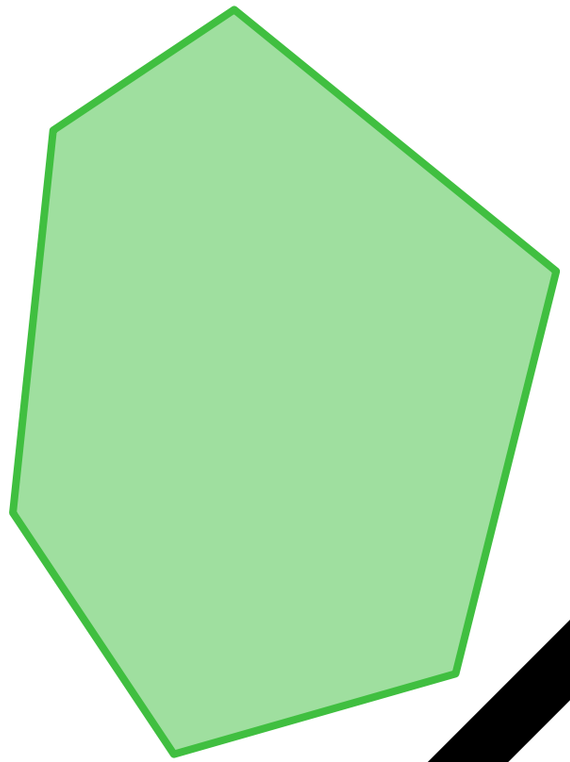

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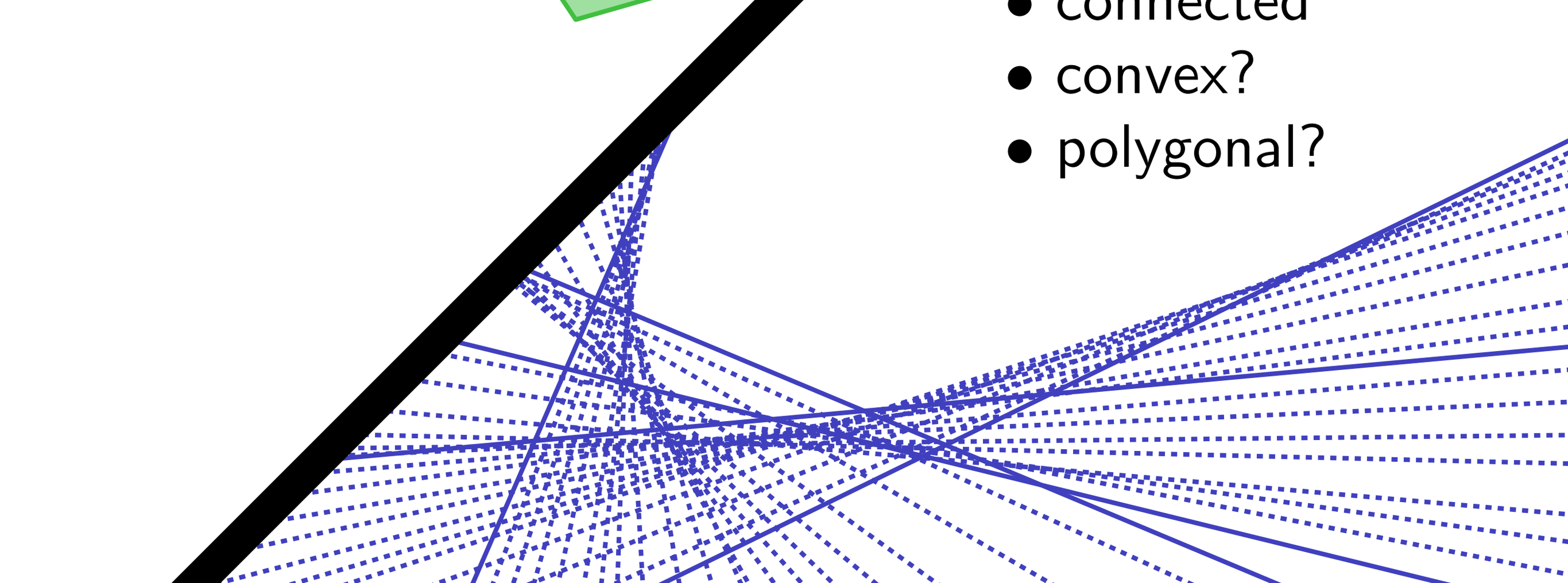

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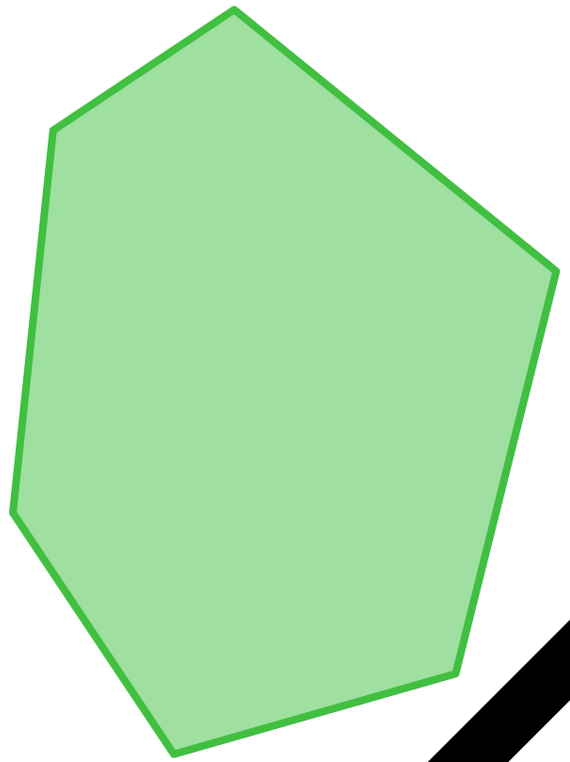

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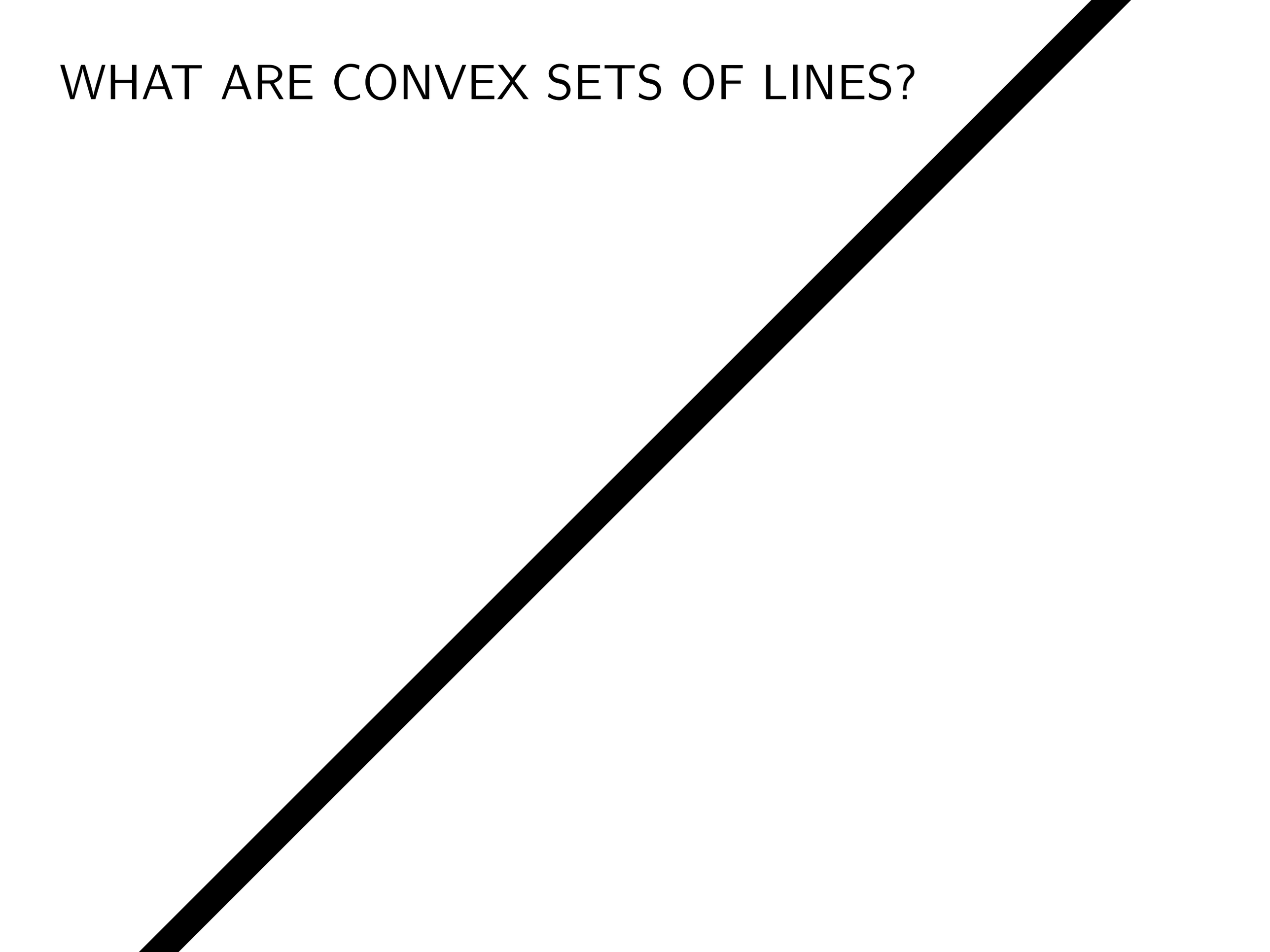

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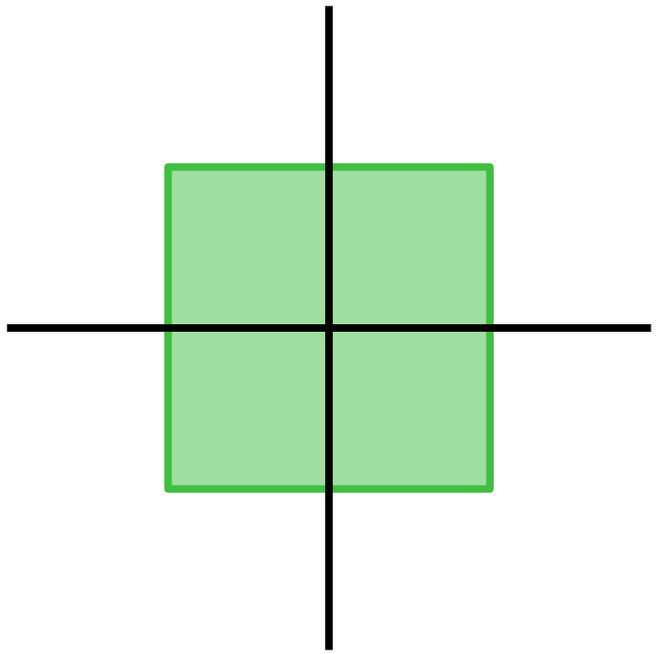

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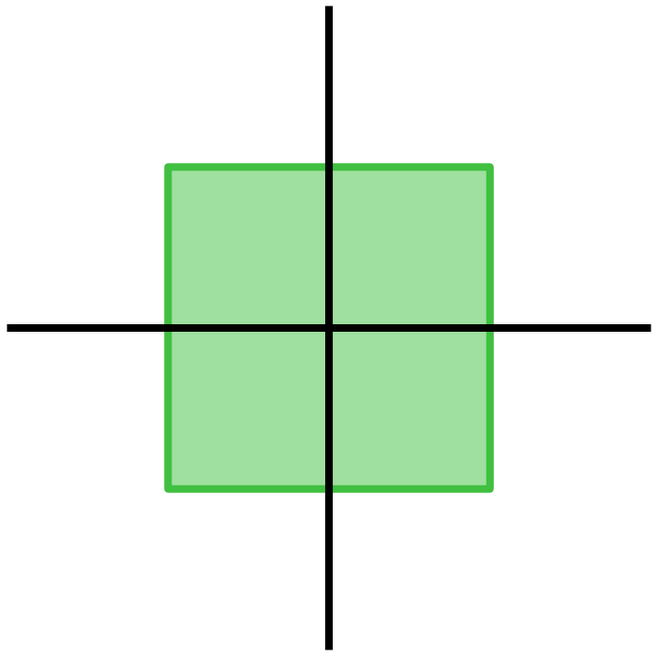

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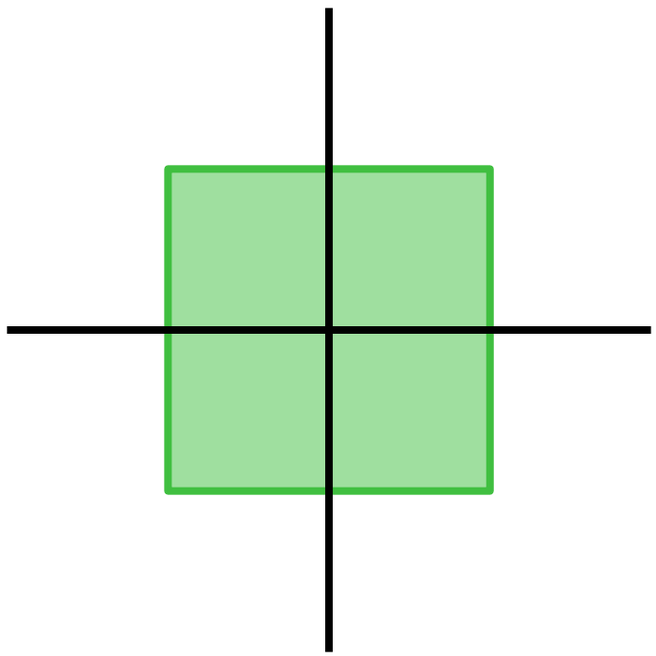

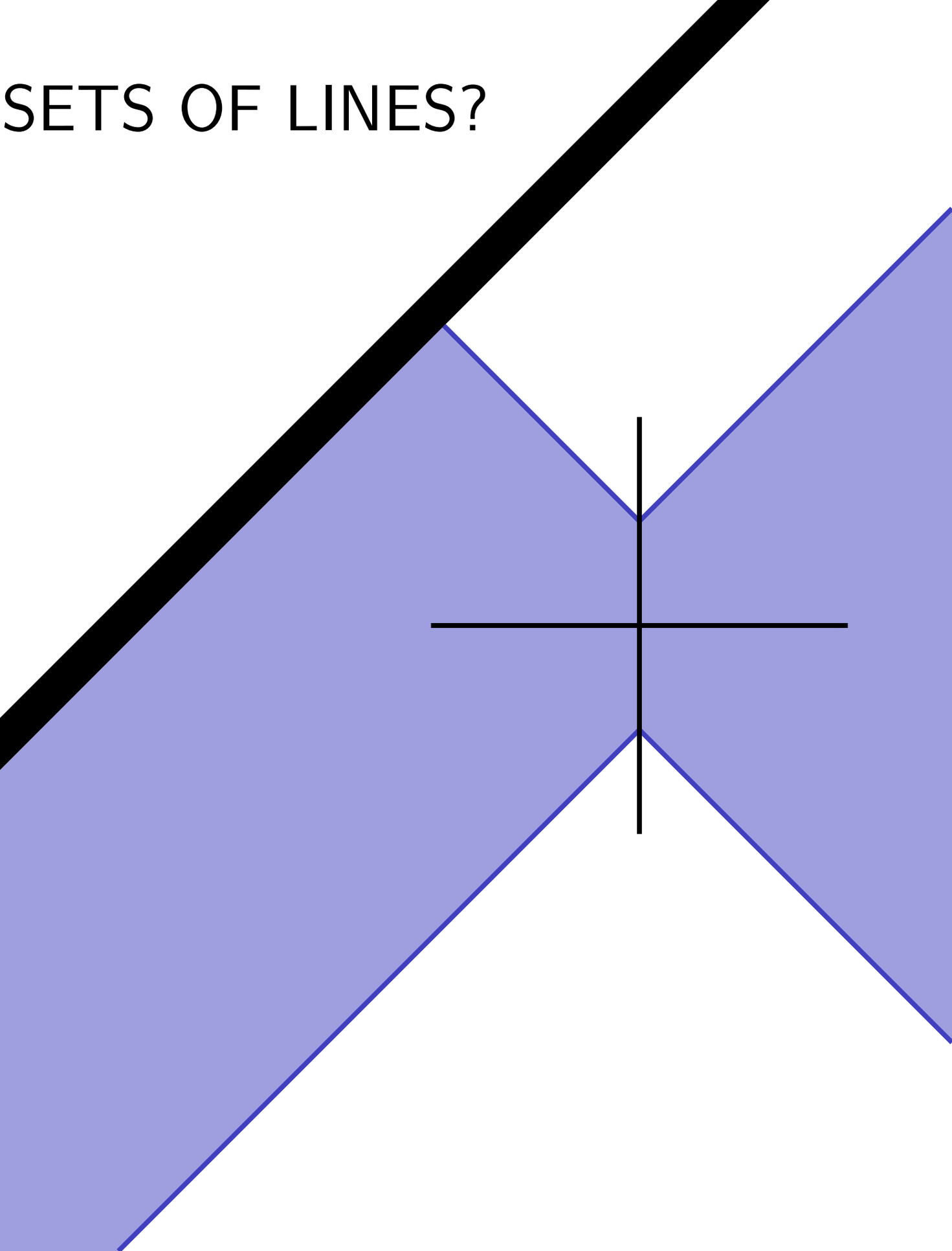

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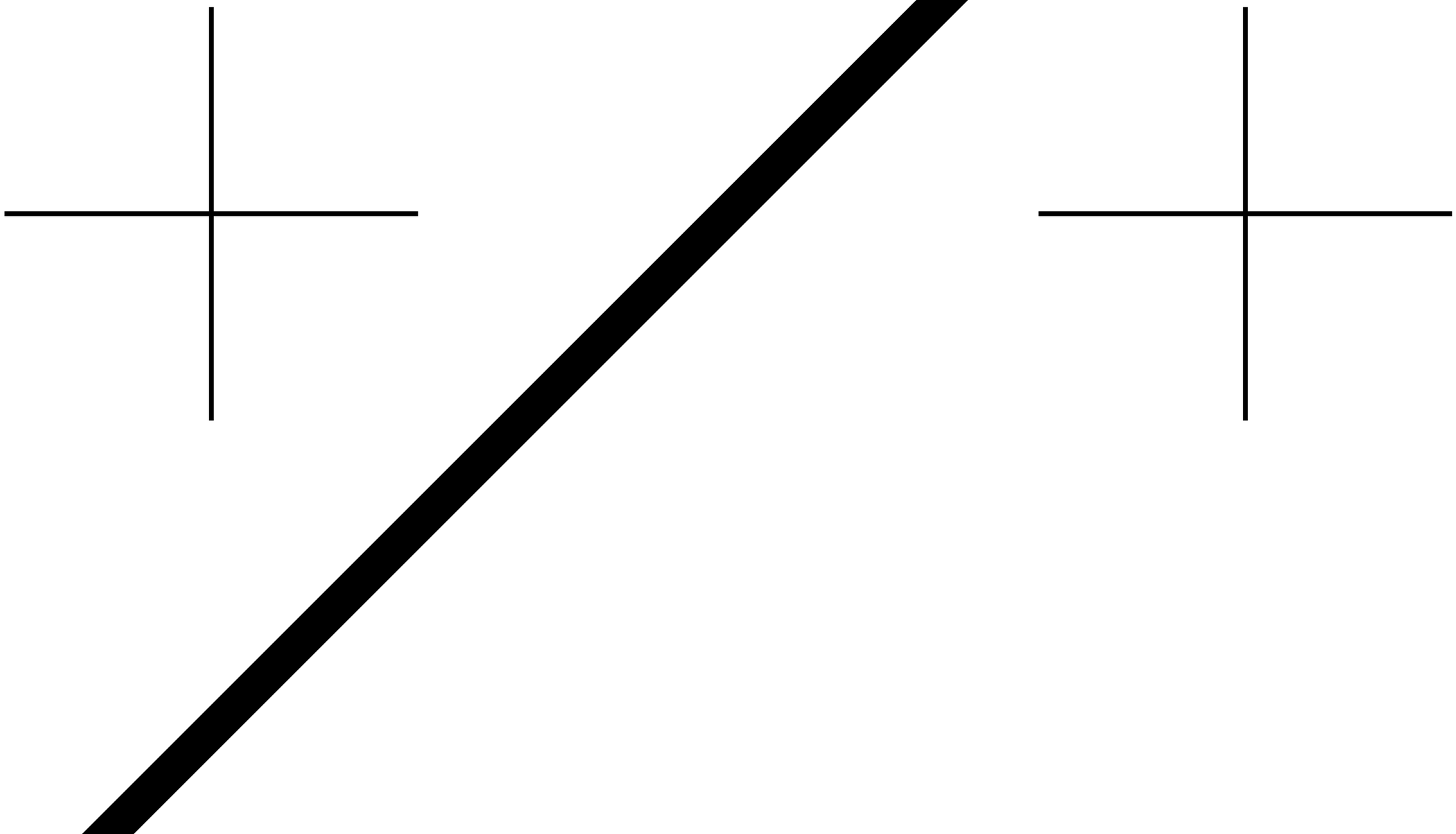

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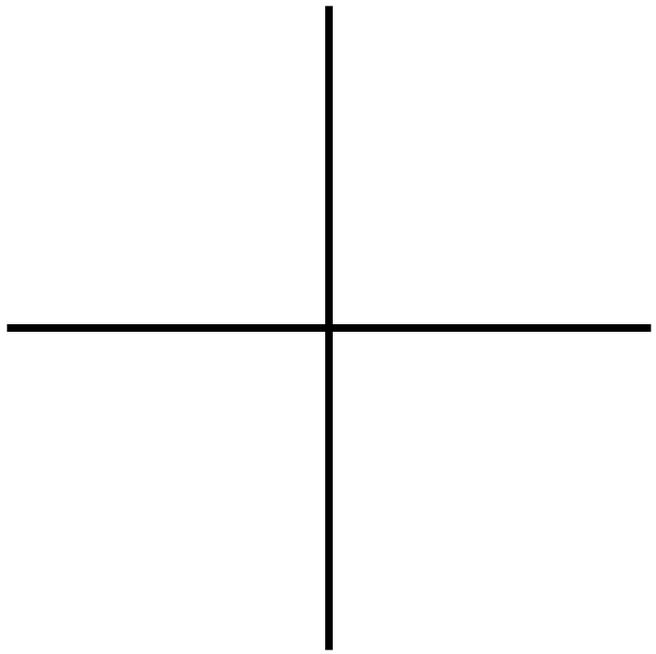

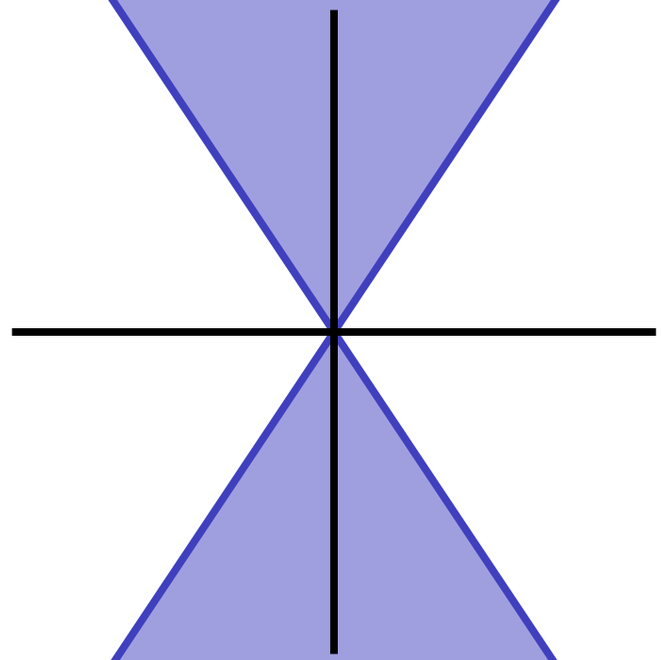

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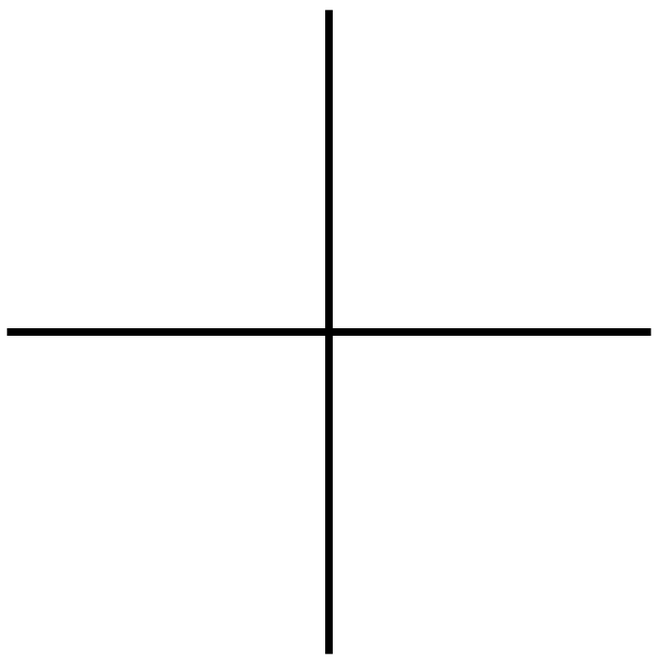

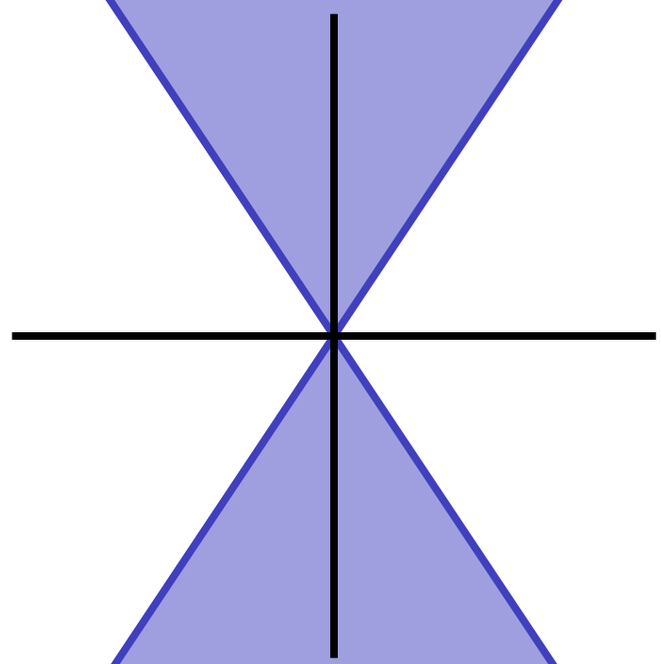

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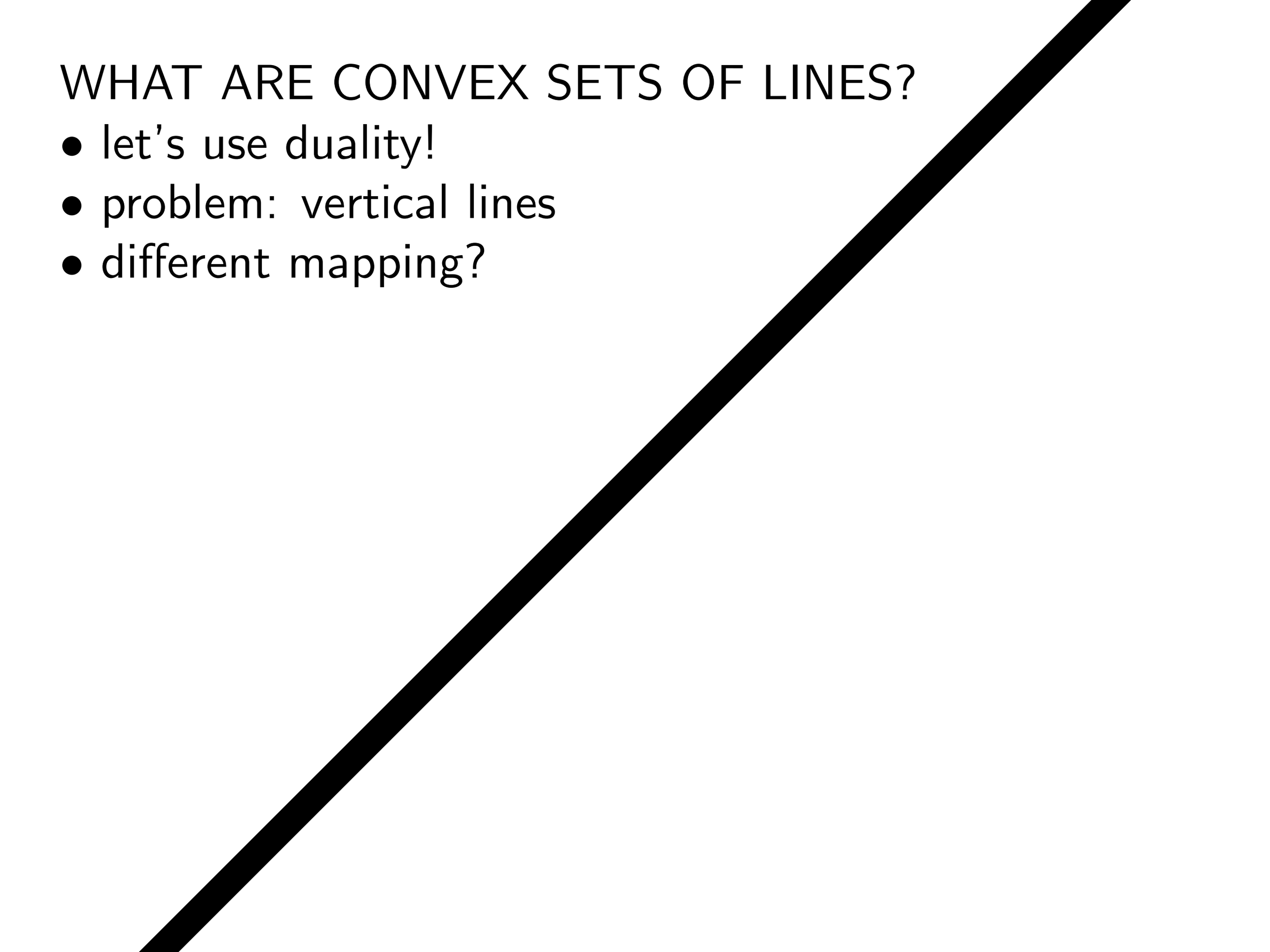

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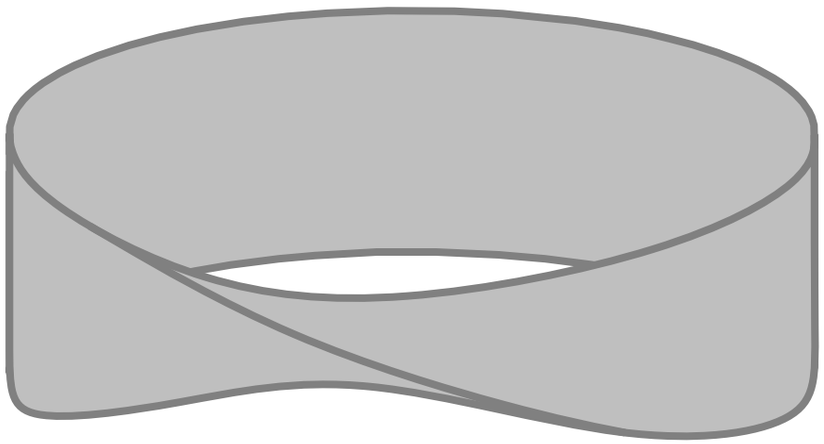

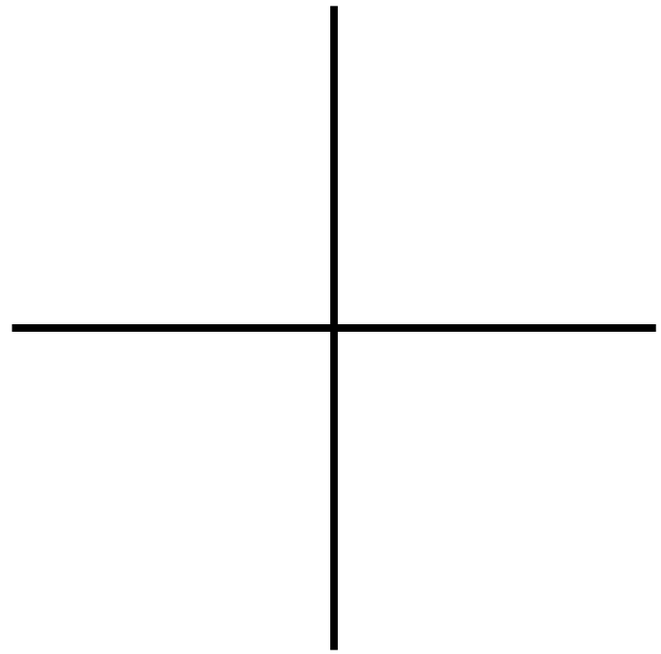

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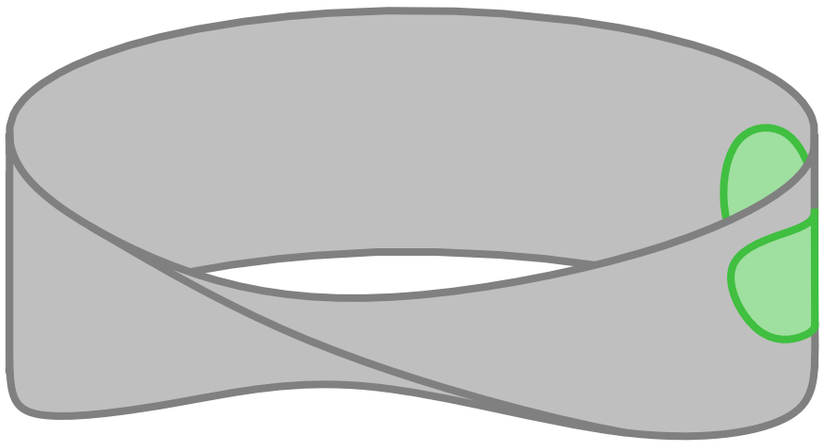

[Rosenfeld, 1995]

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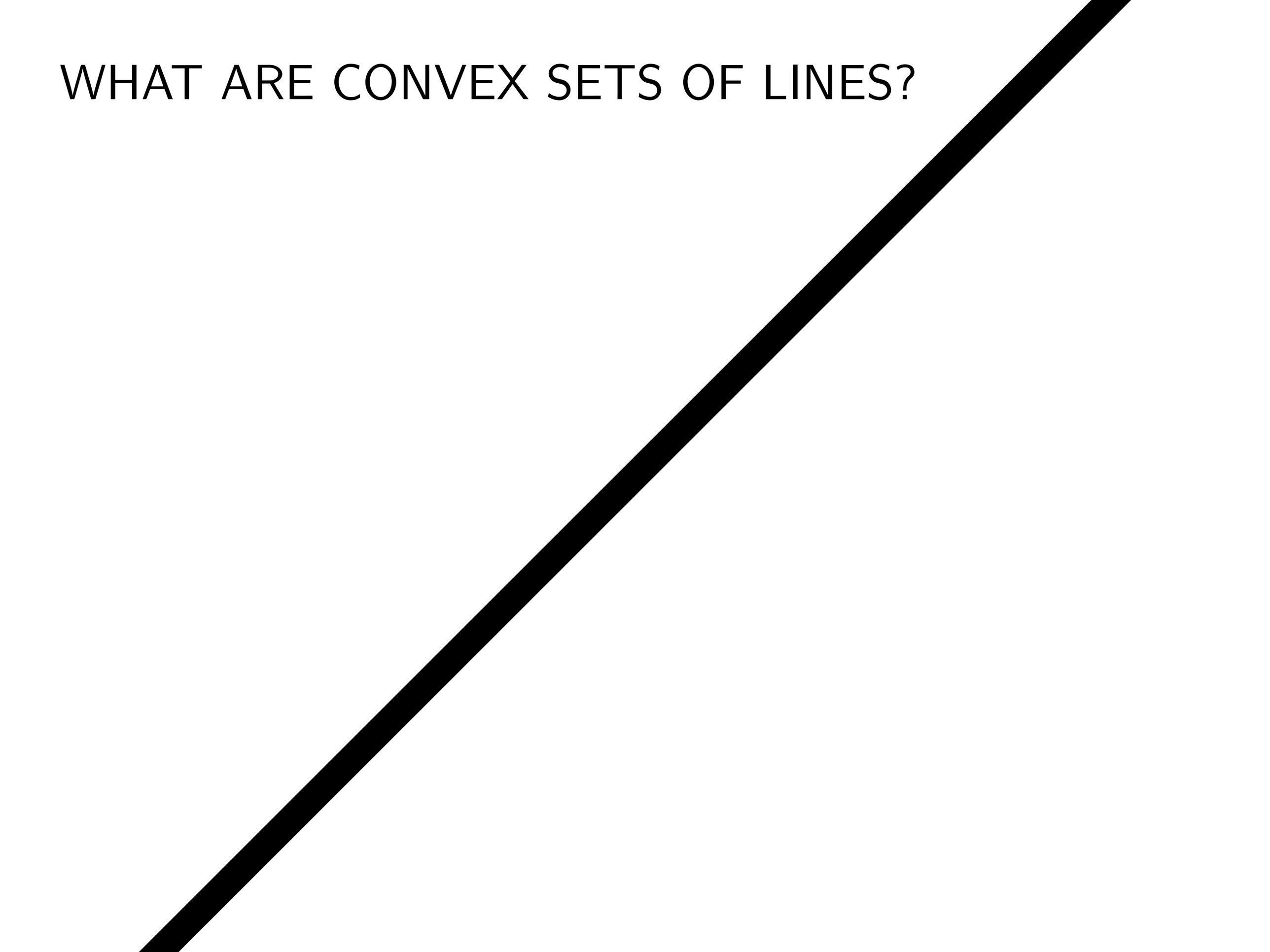

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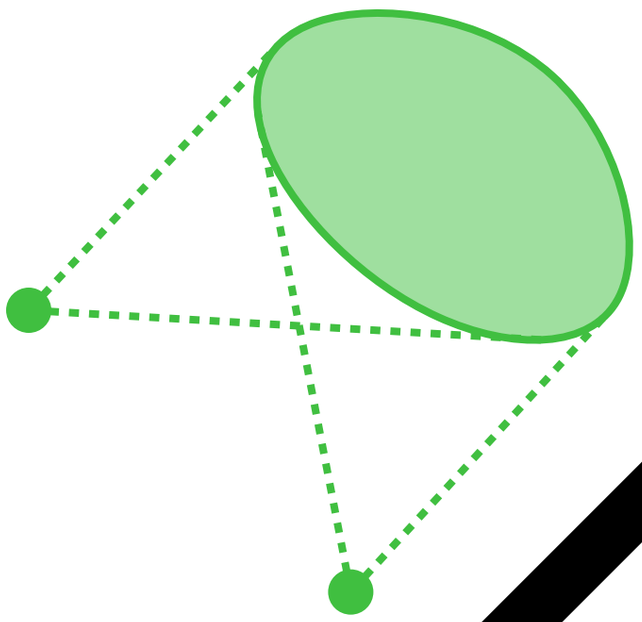

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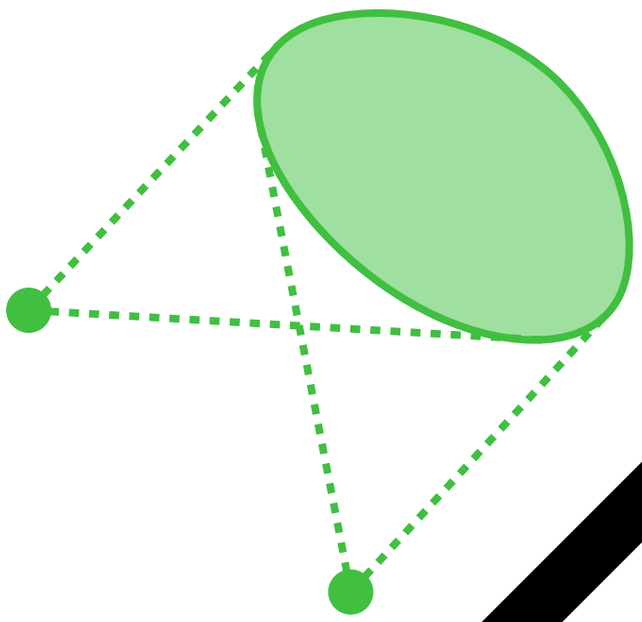

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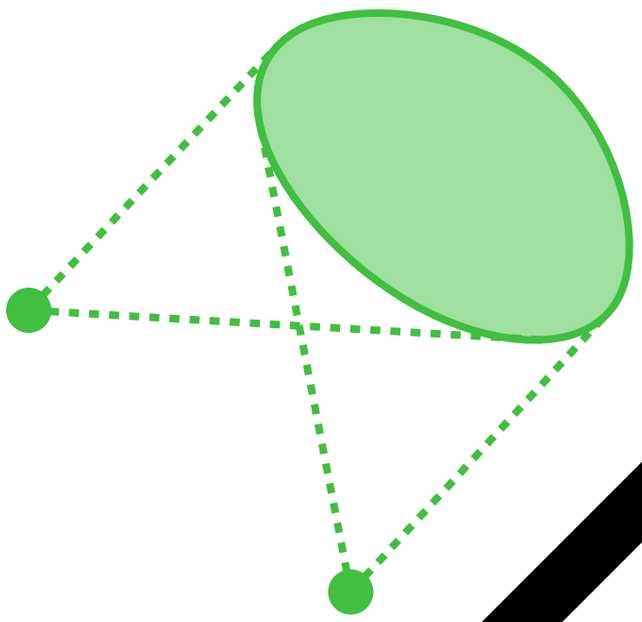

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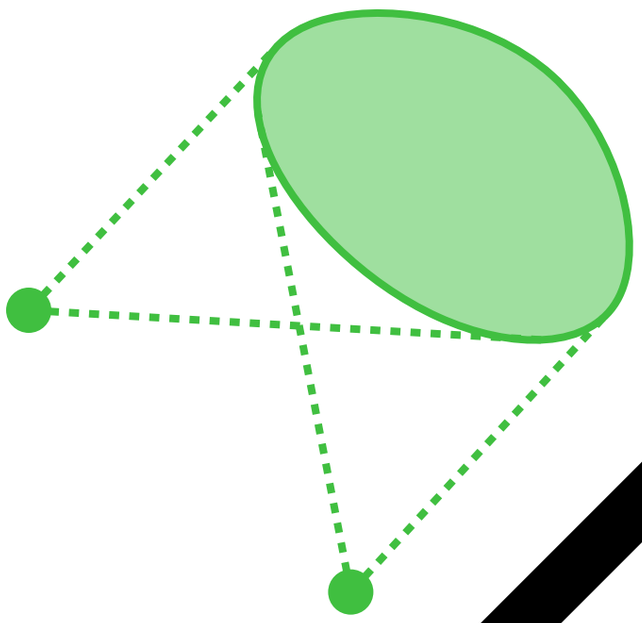

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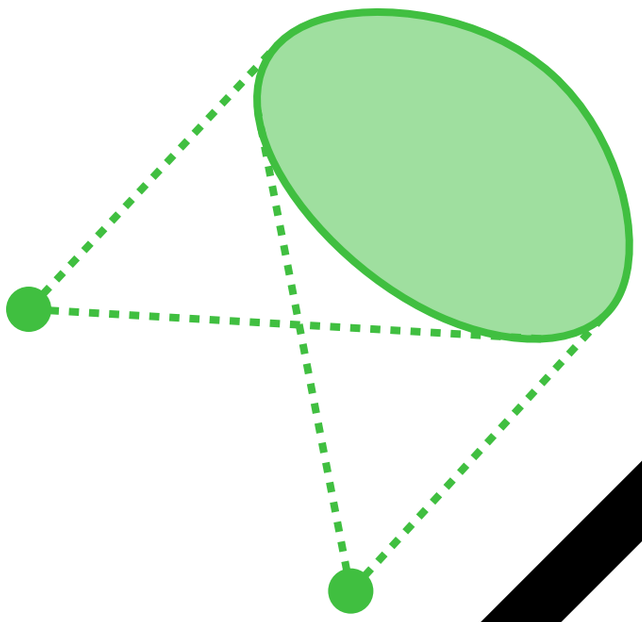

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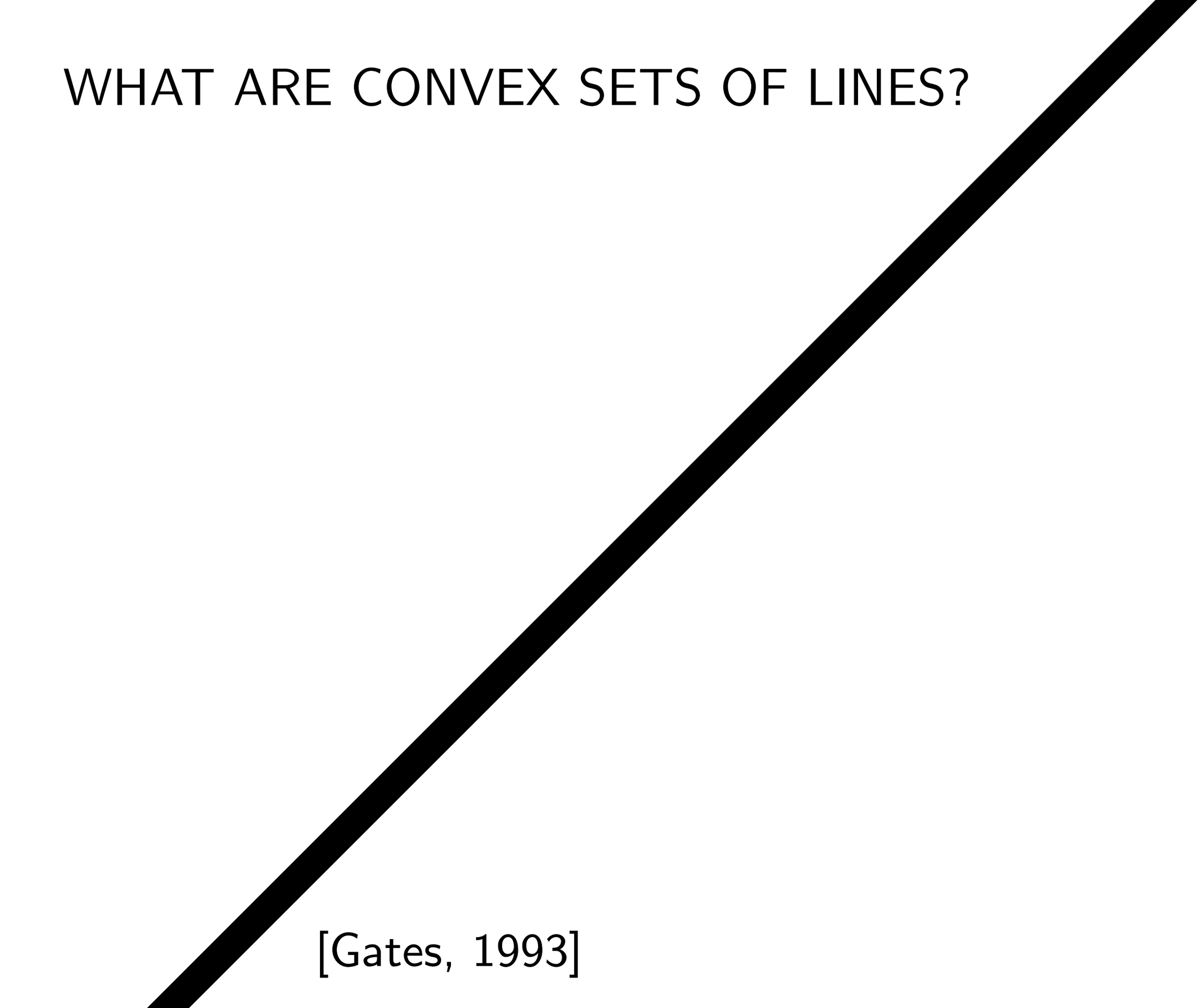

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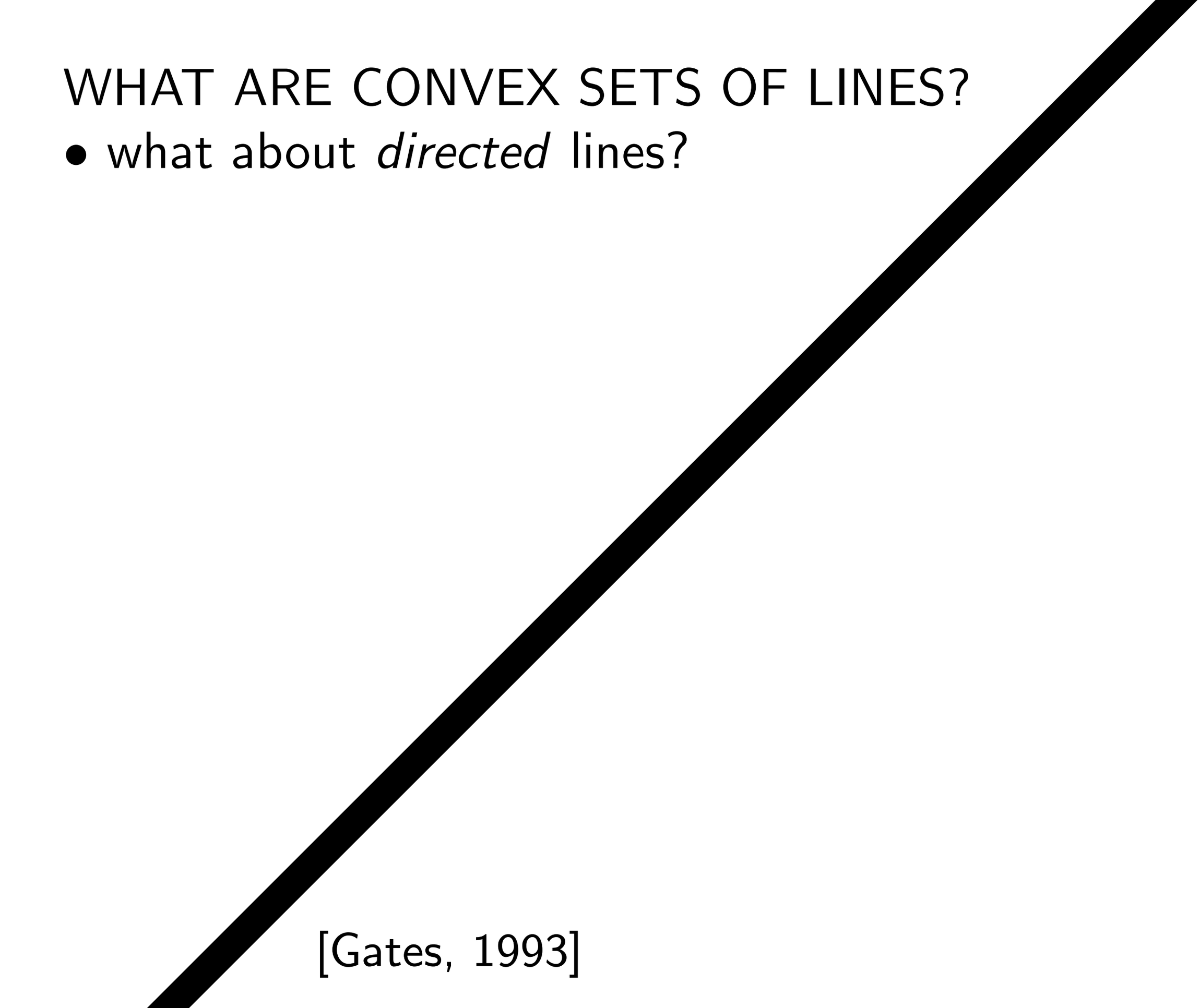

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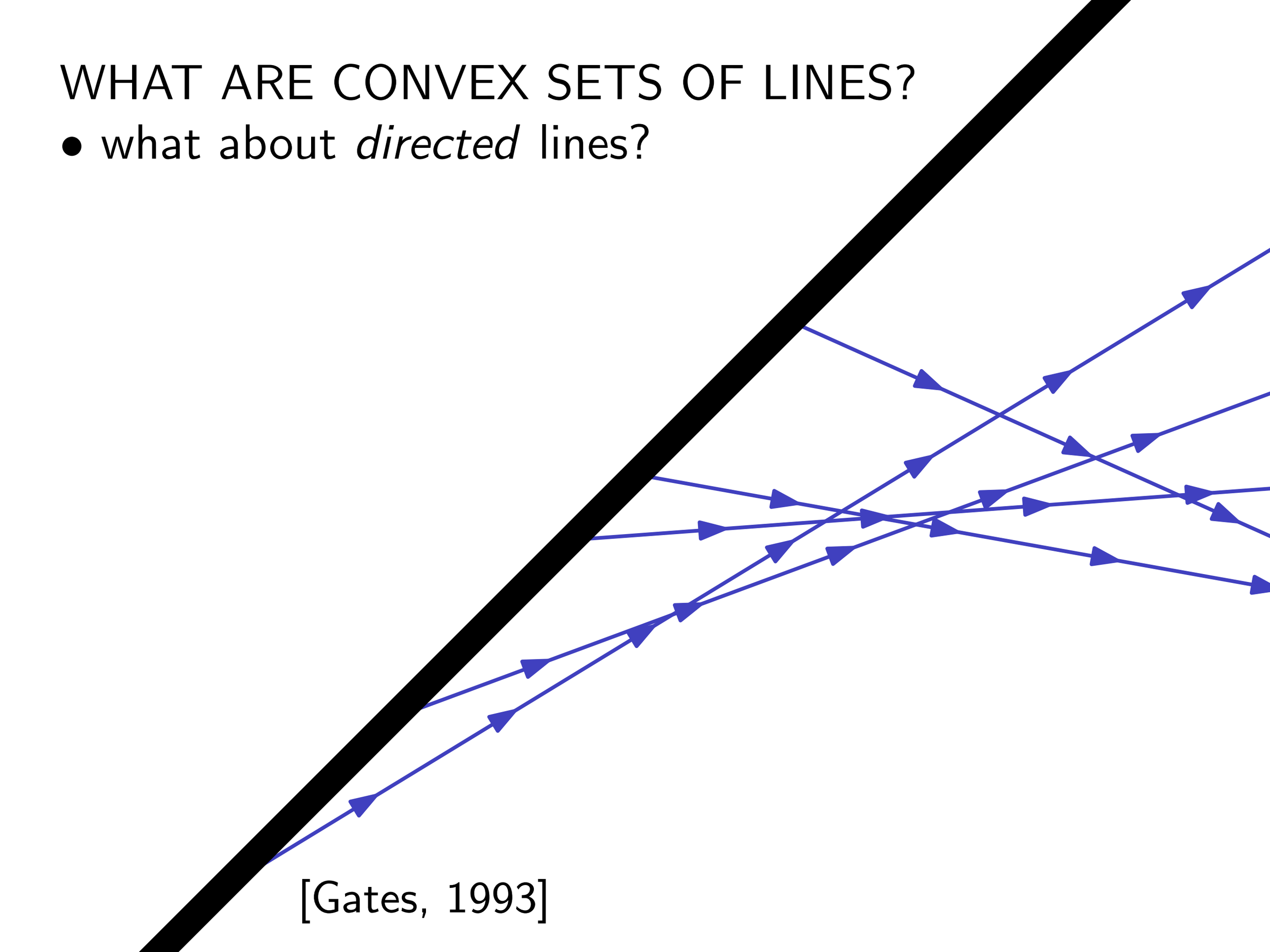

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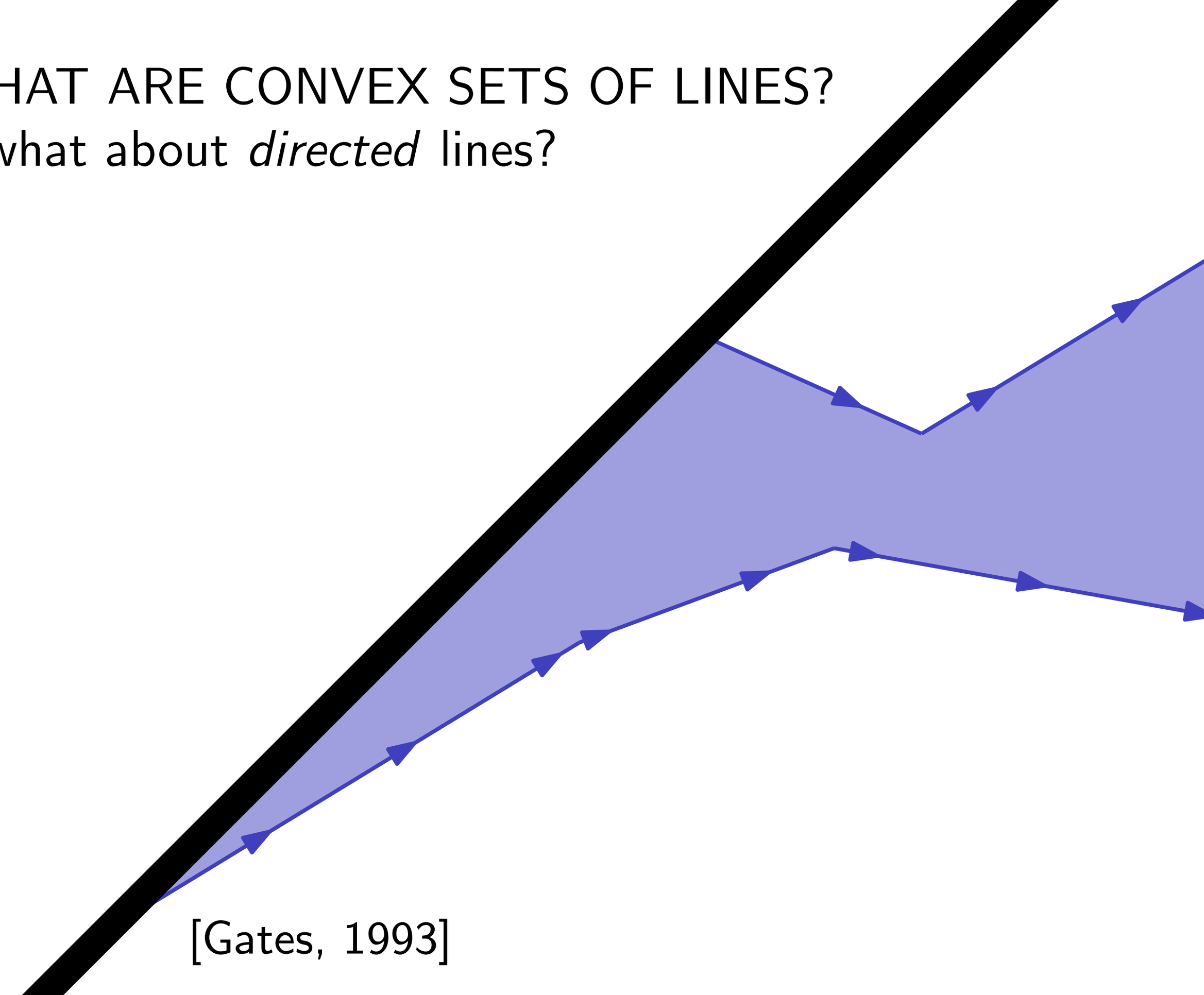

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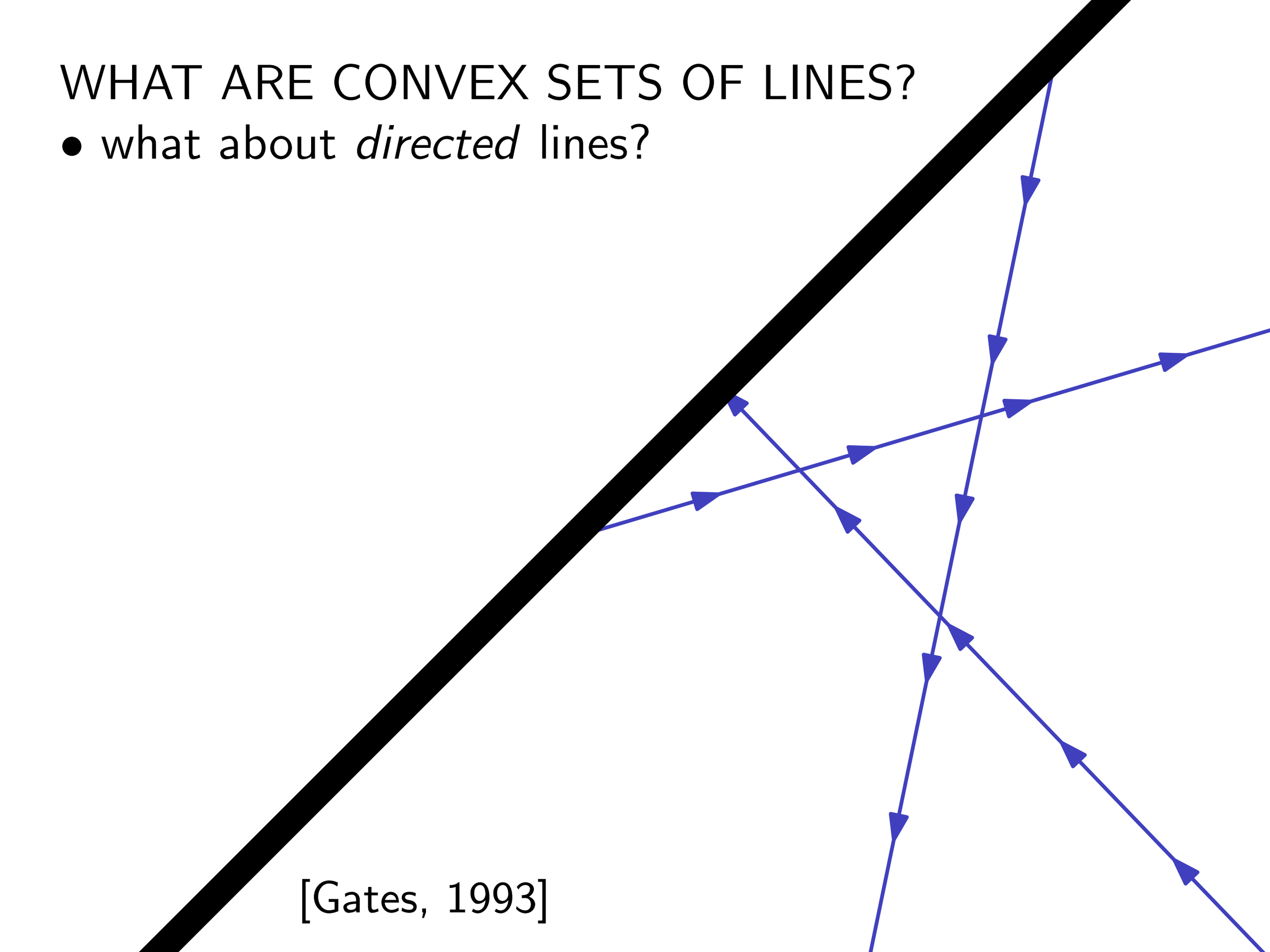

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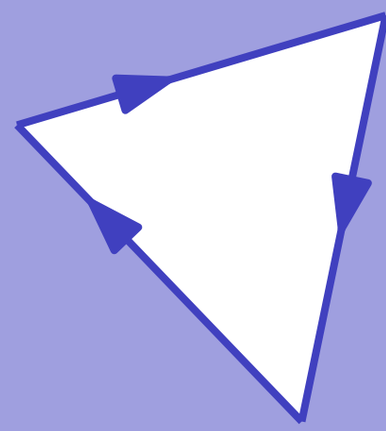

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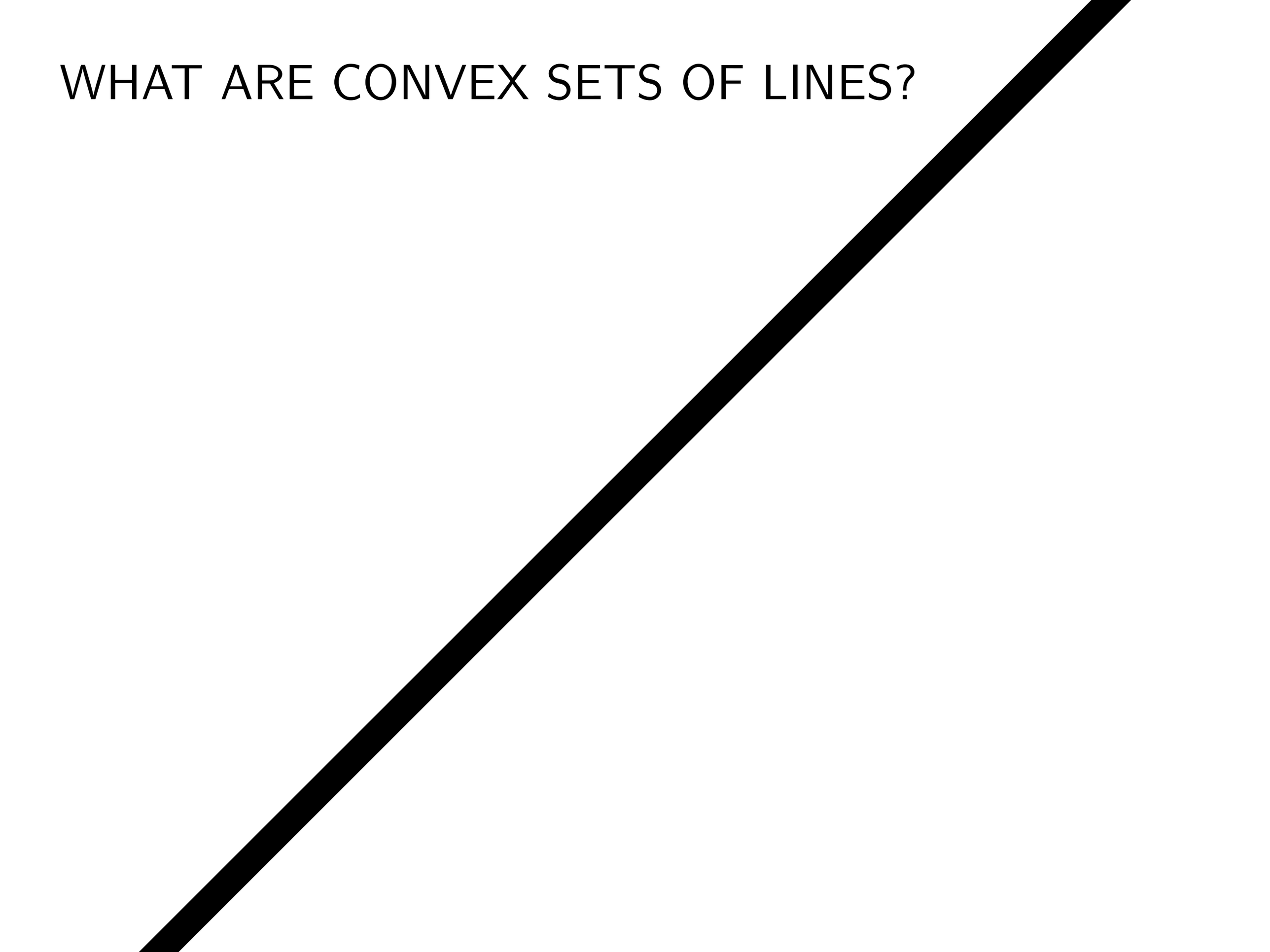

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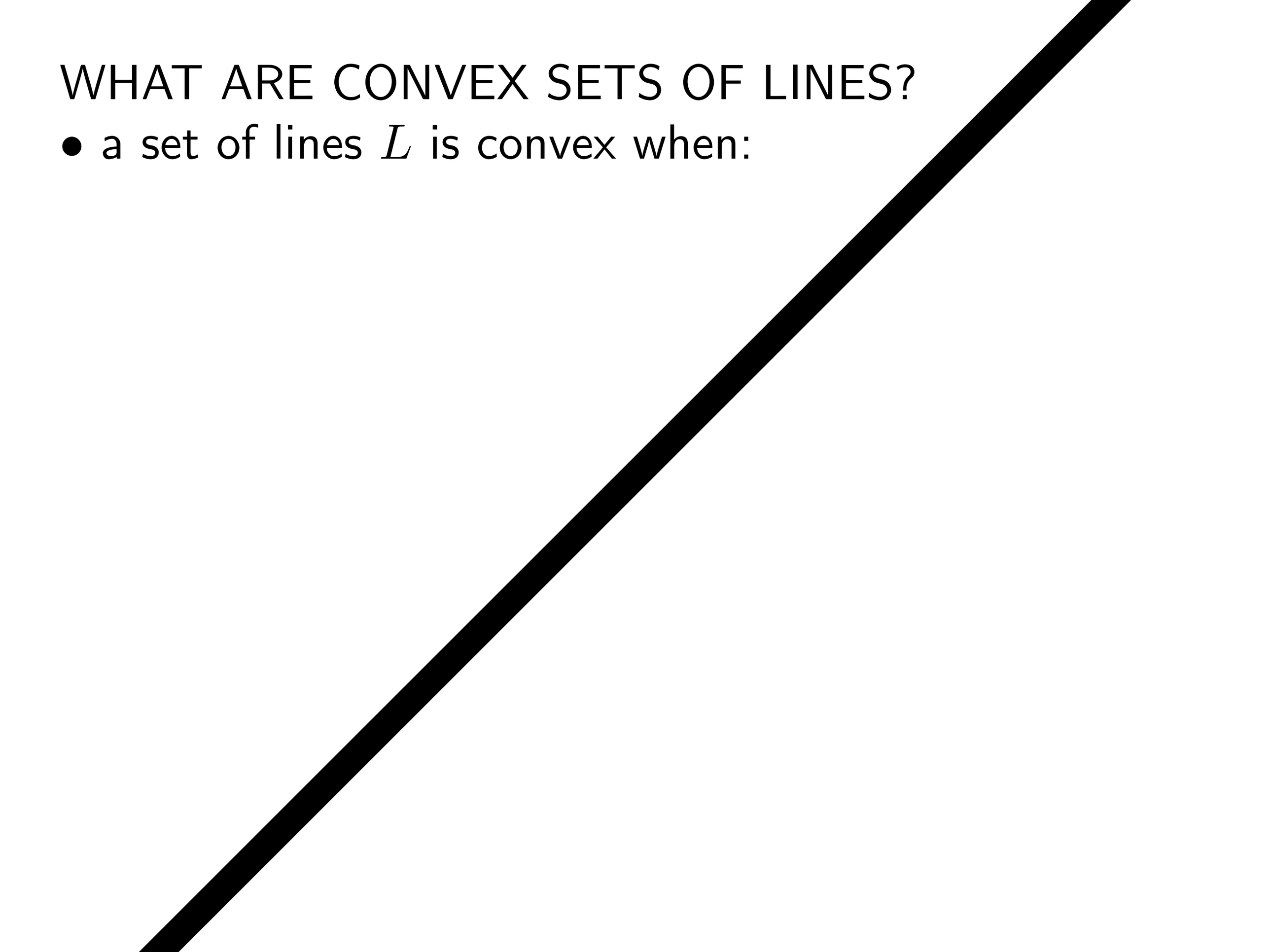

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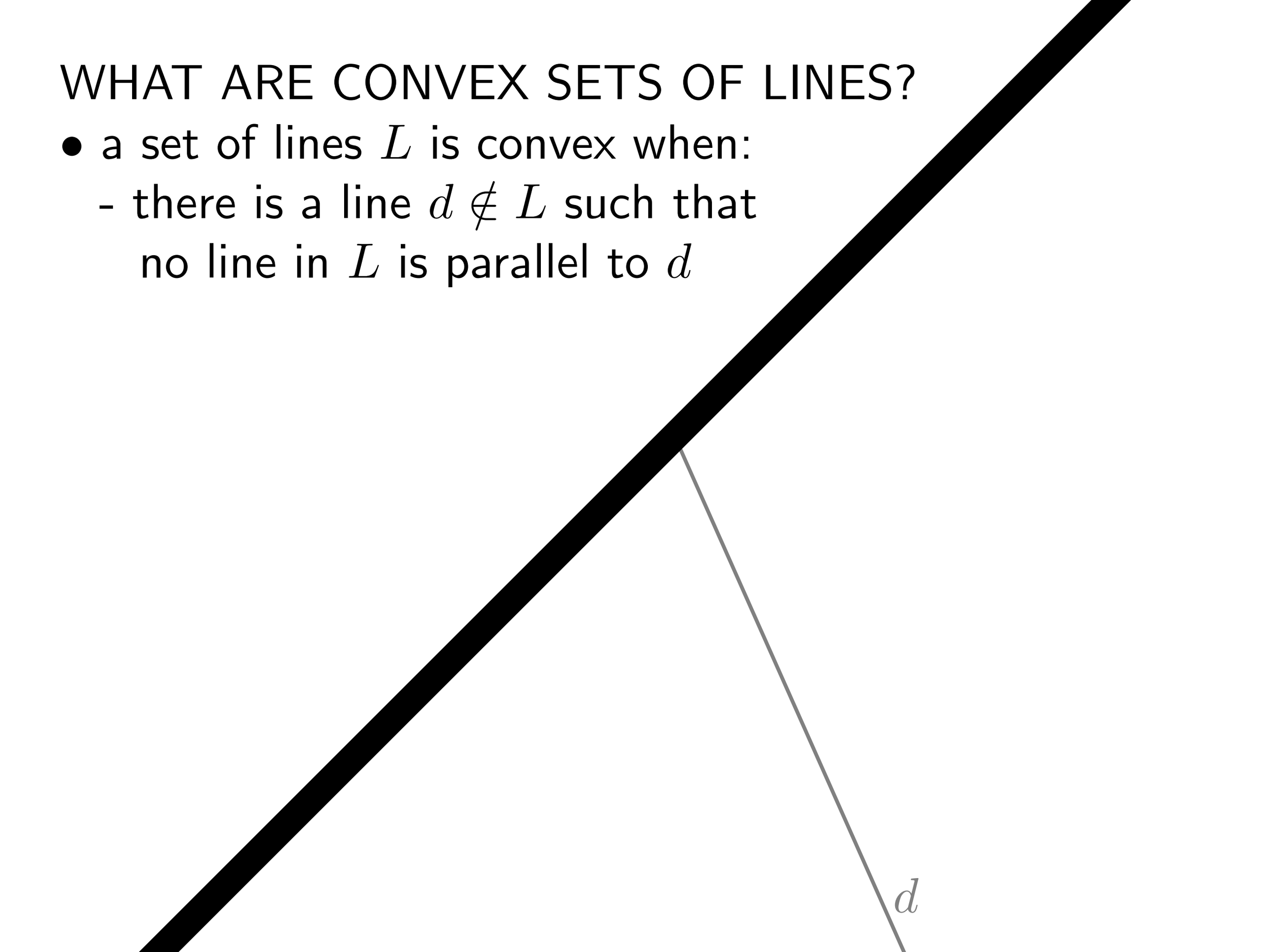

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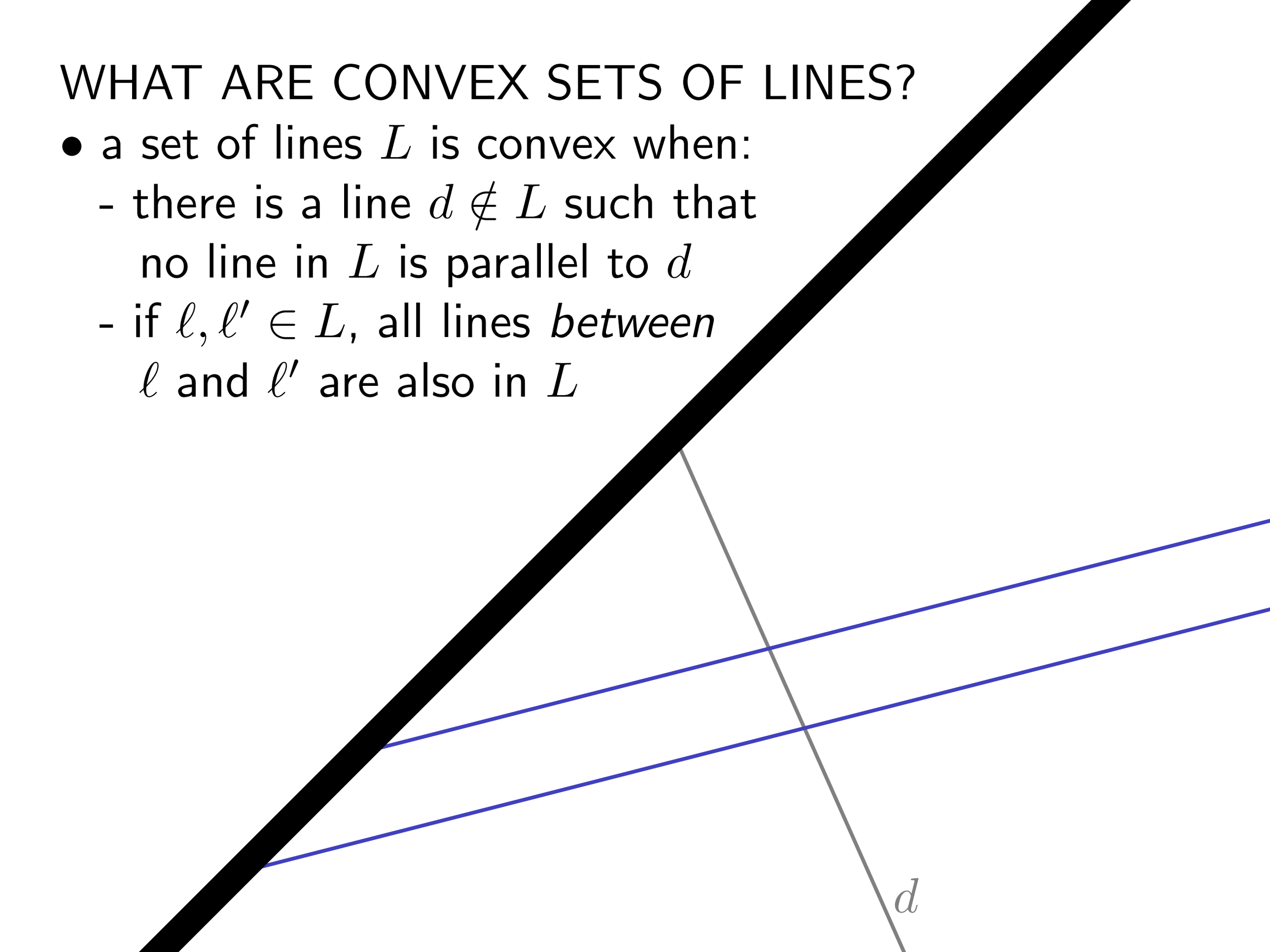

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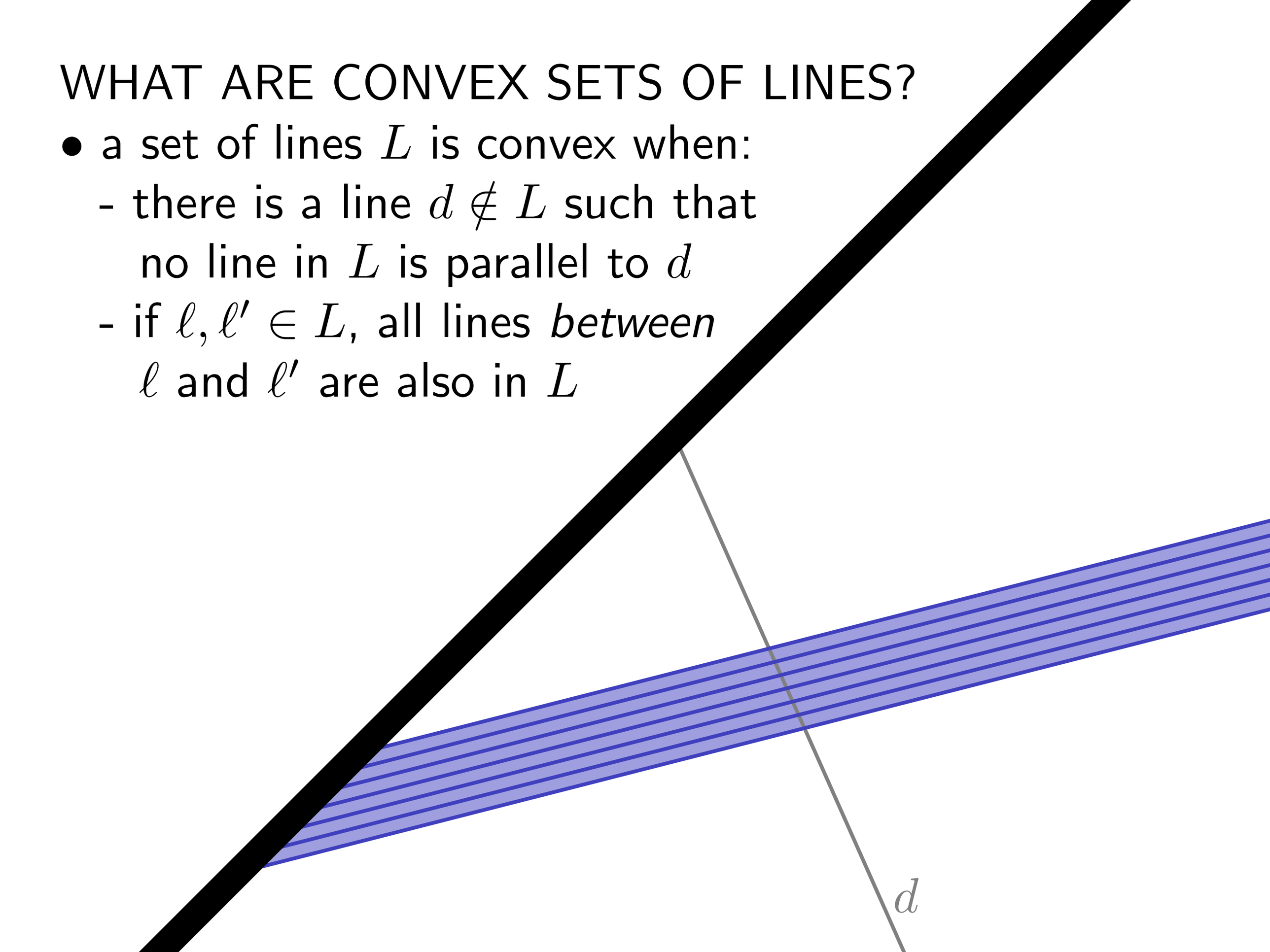

WHAT ARE CONVEX SETS OF LINES?

- a set of lines L is convex when:
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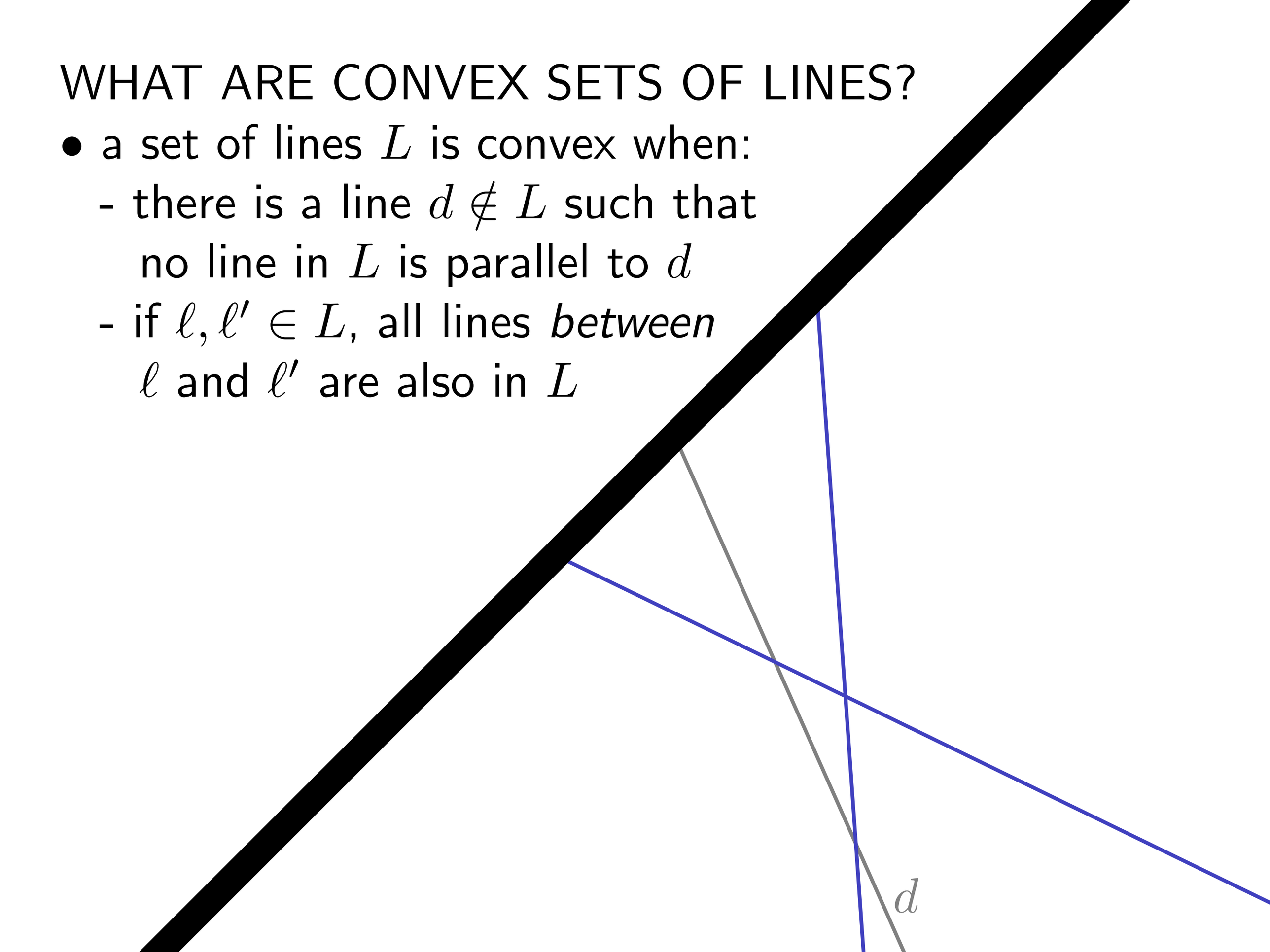

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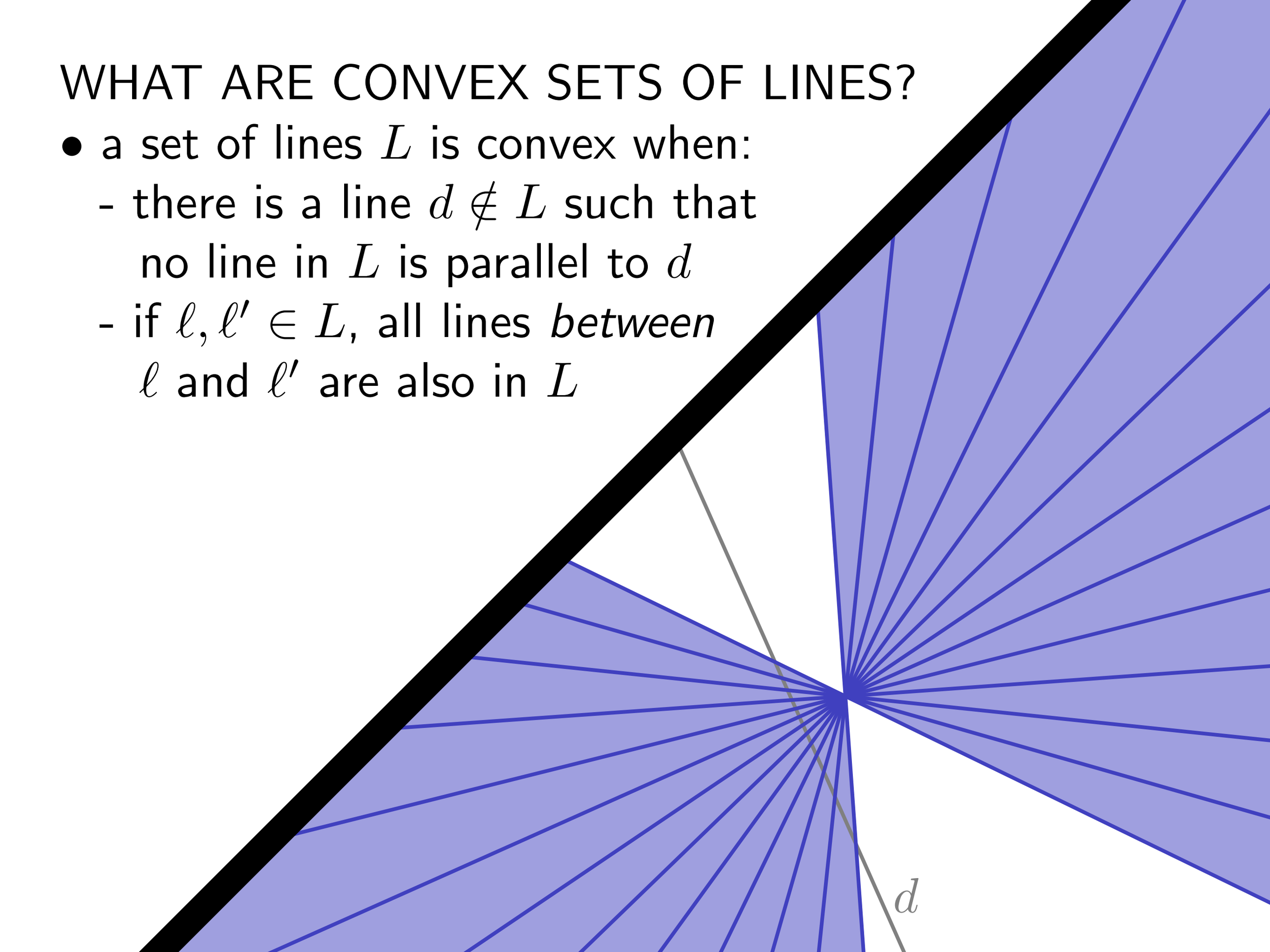

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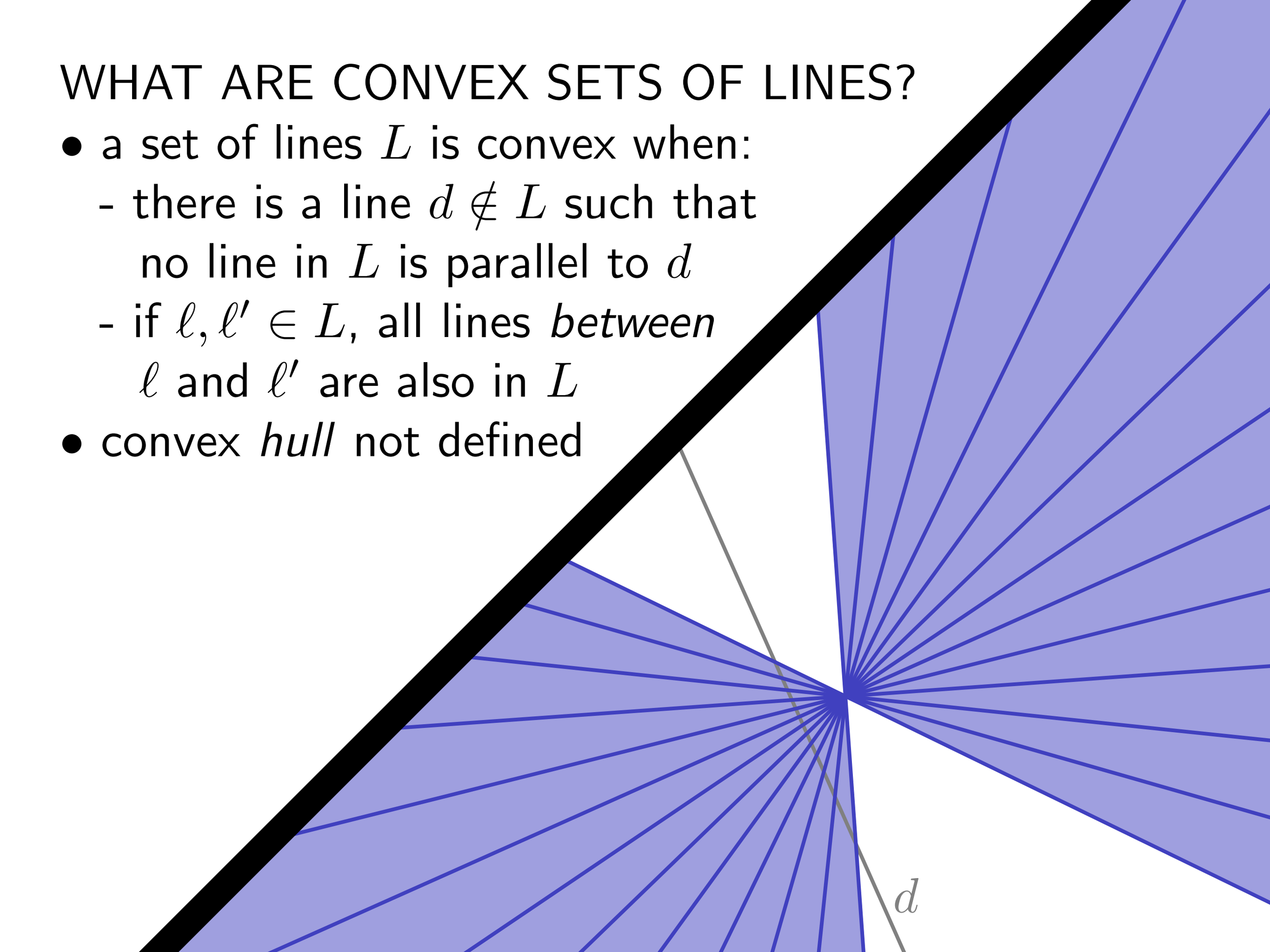

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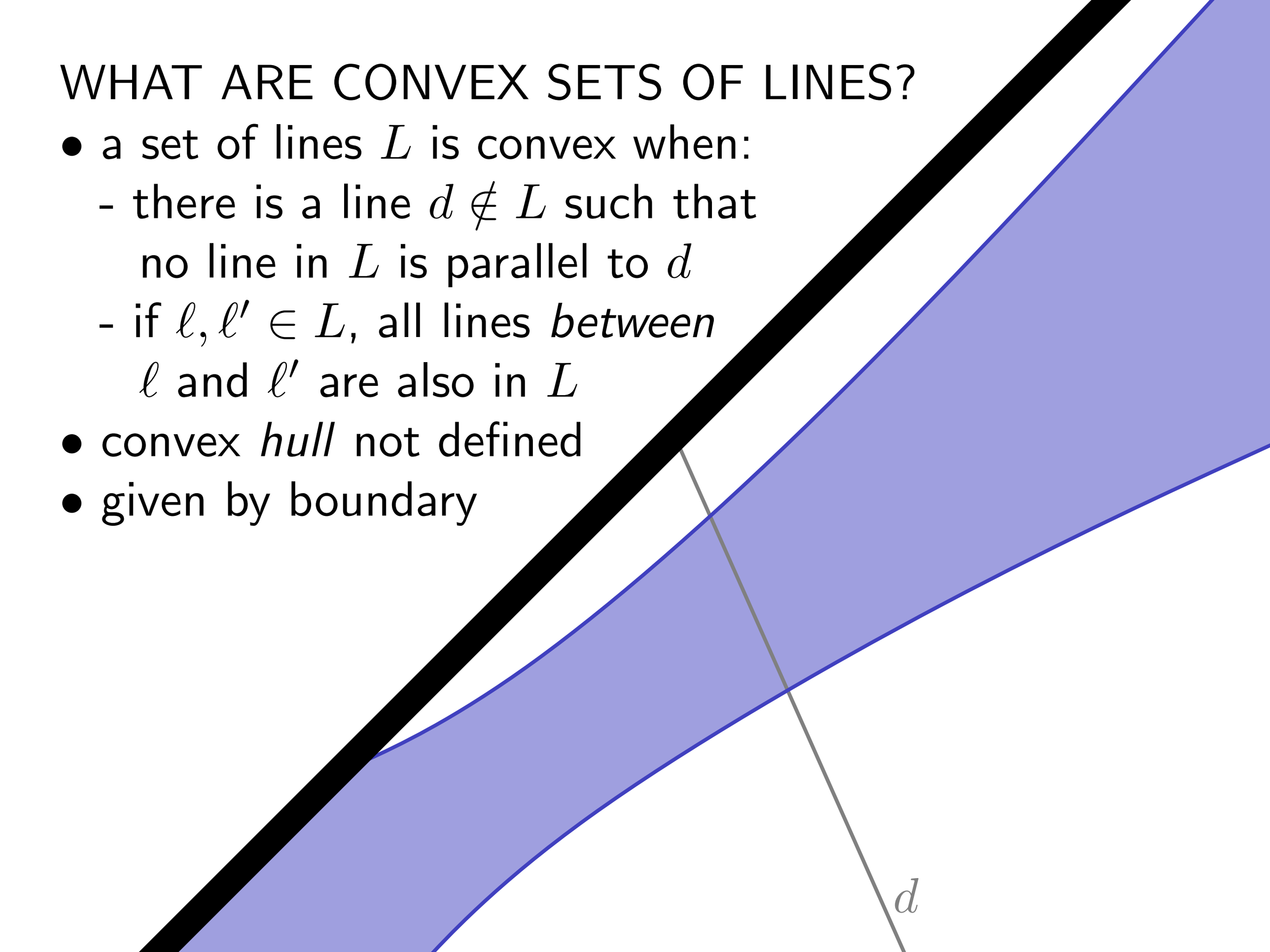

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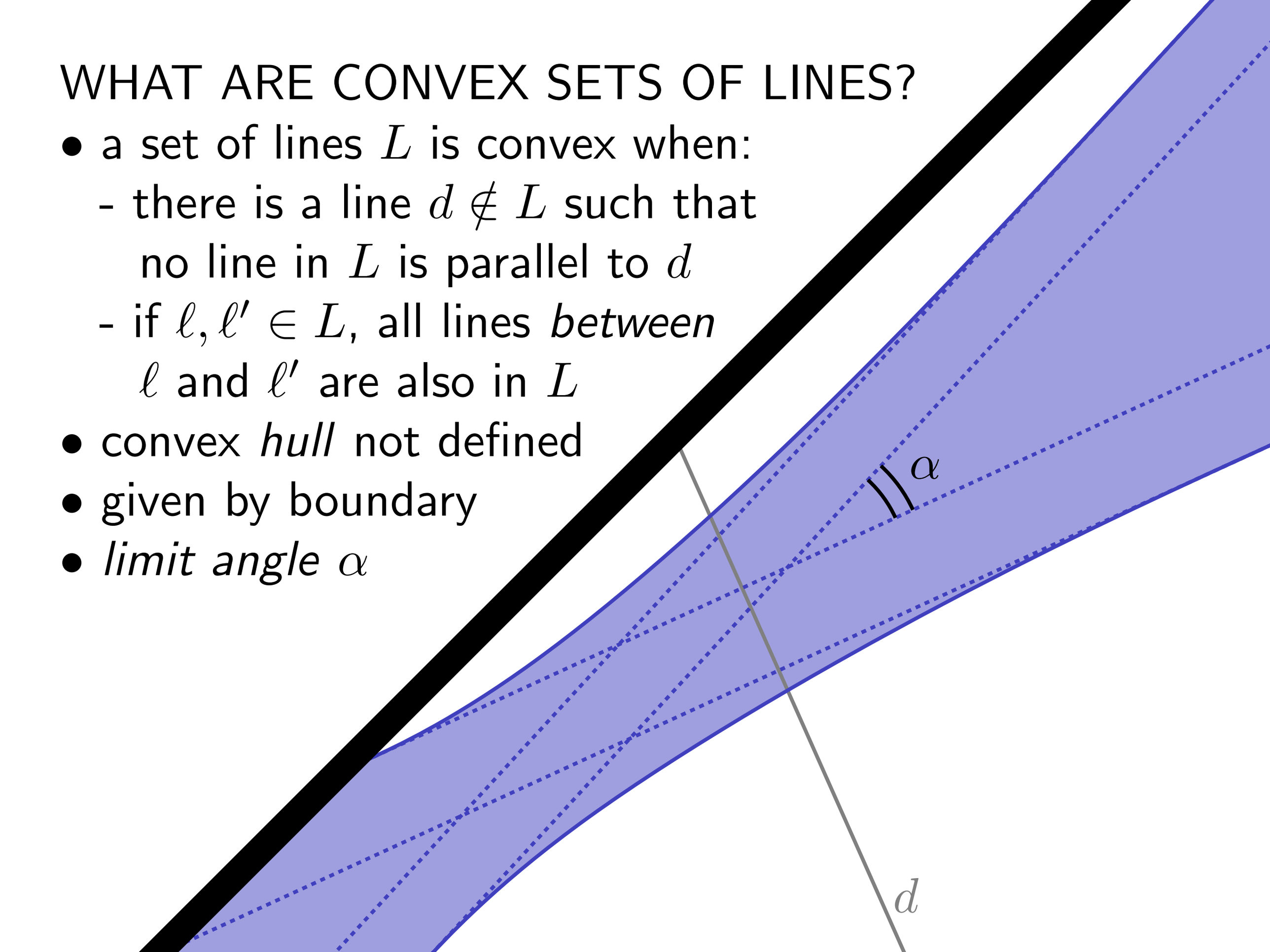
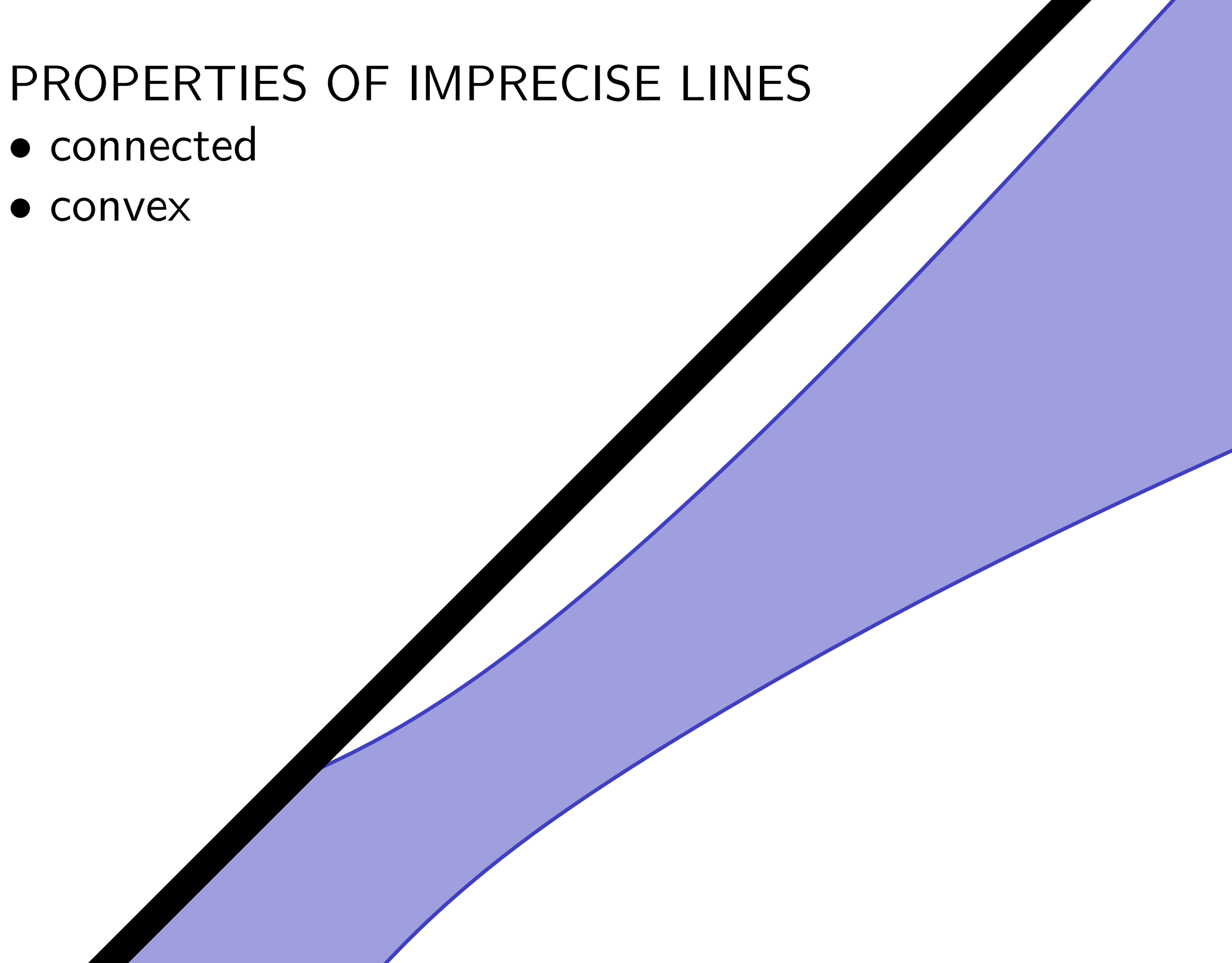
- *limit angle* α



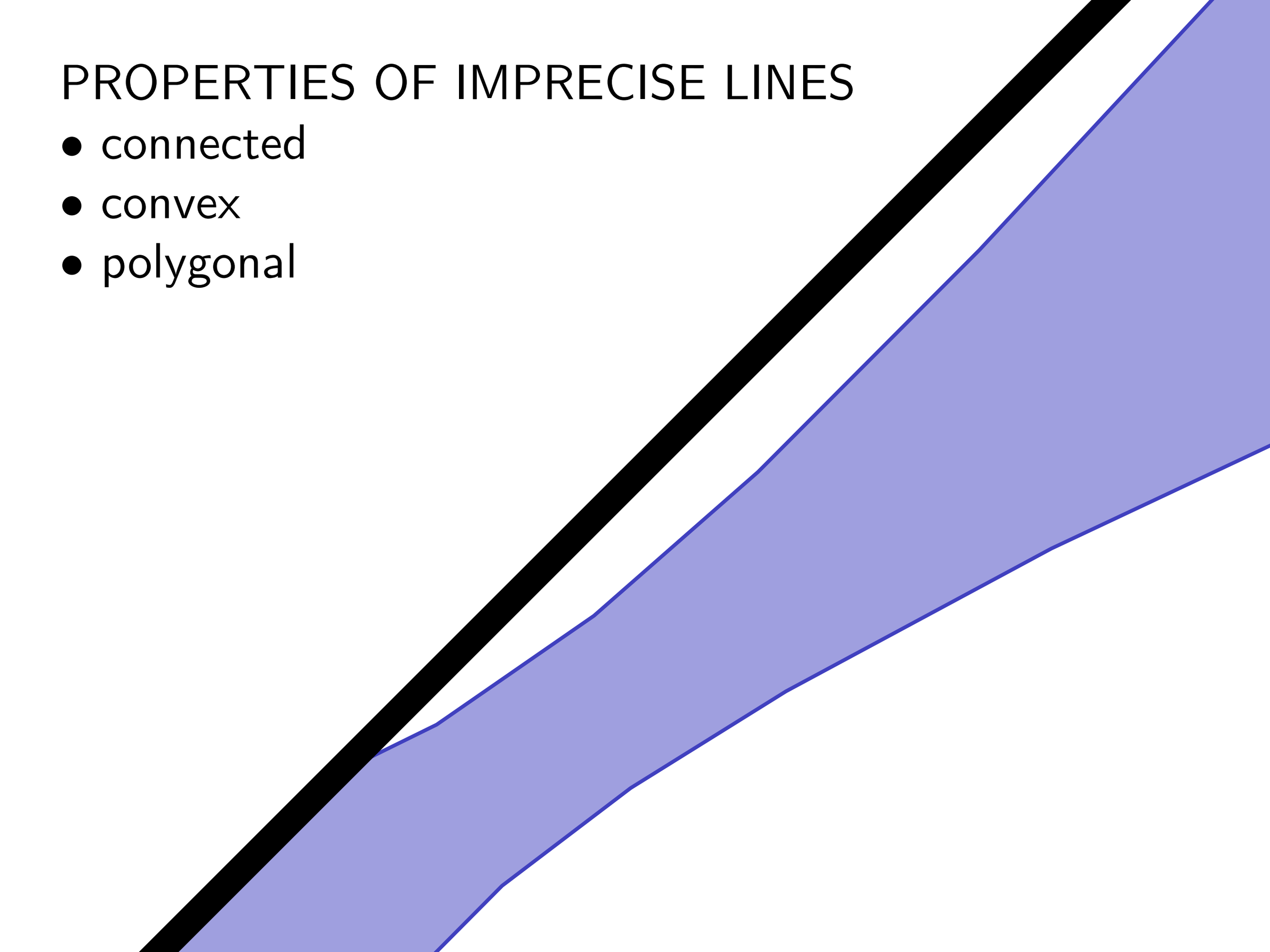
PROPERTIES OF IMPRECISE LINES

- connected
- convex



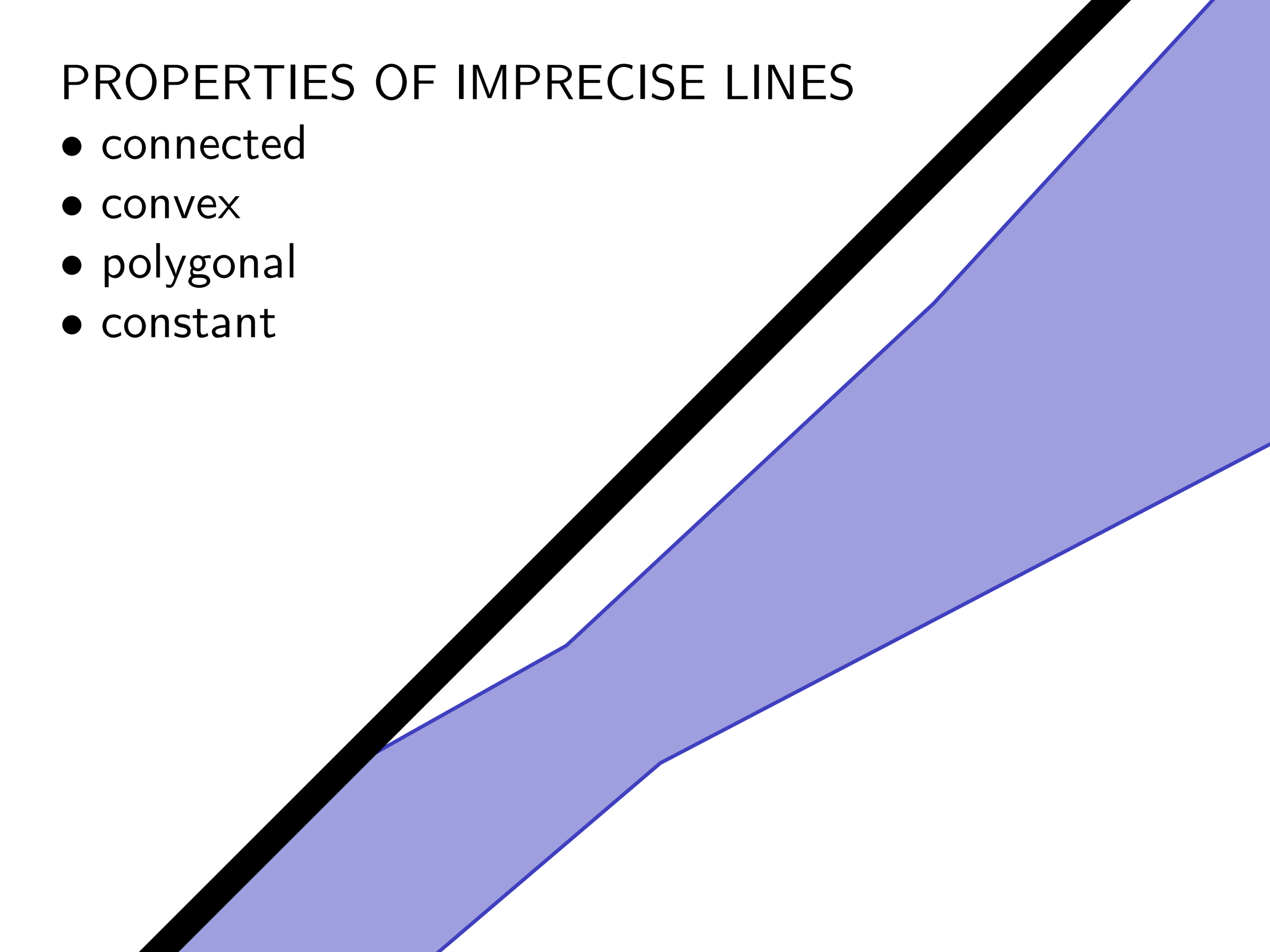
PROPERTIES OF IMPRECISE LINES

- connected
- convex
- polygonal



PROPERTIES OF IMPRECISE LINES

- connected
- convex
- polygonal
- constant



EXAMPLE: LINEAR PROGRAMMING

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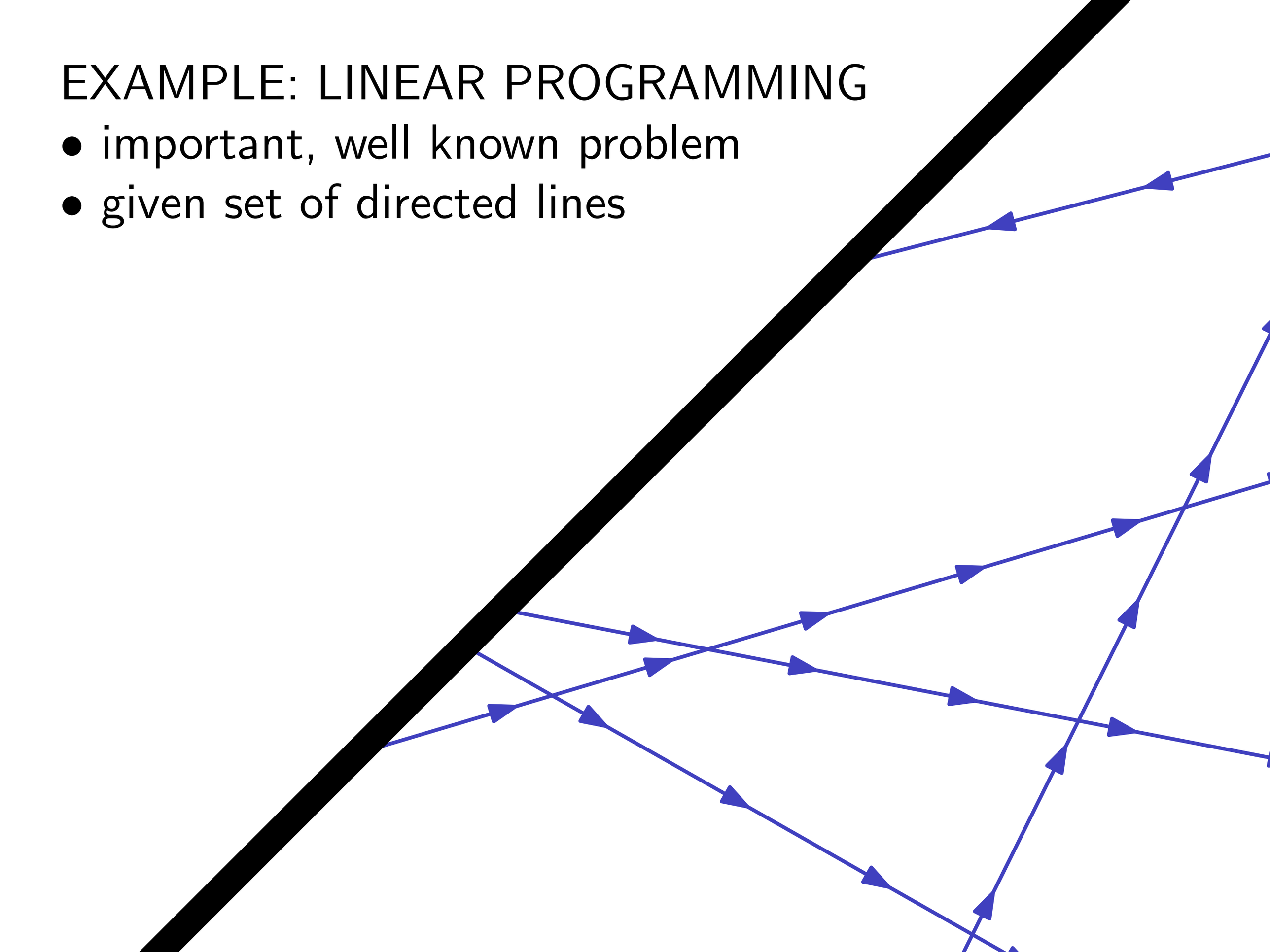
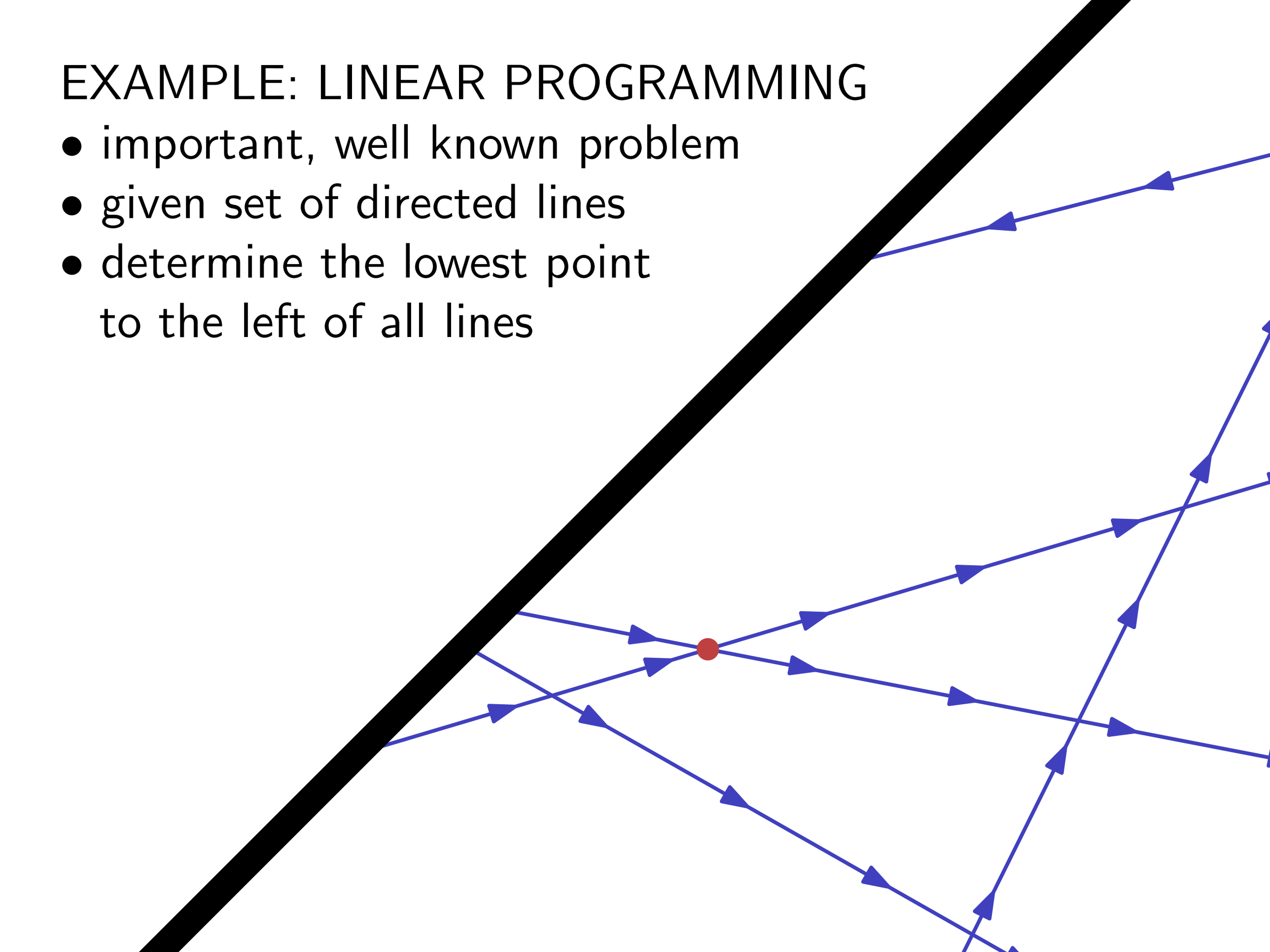


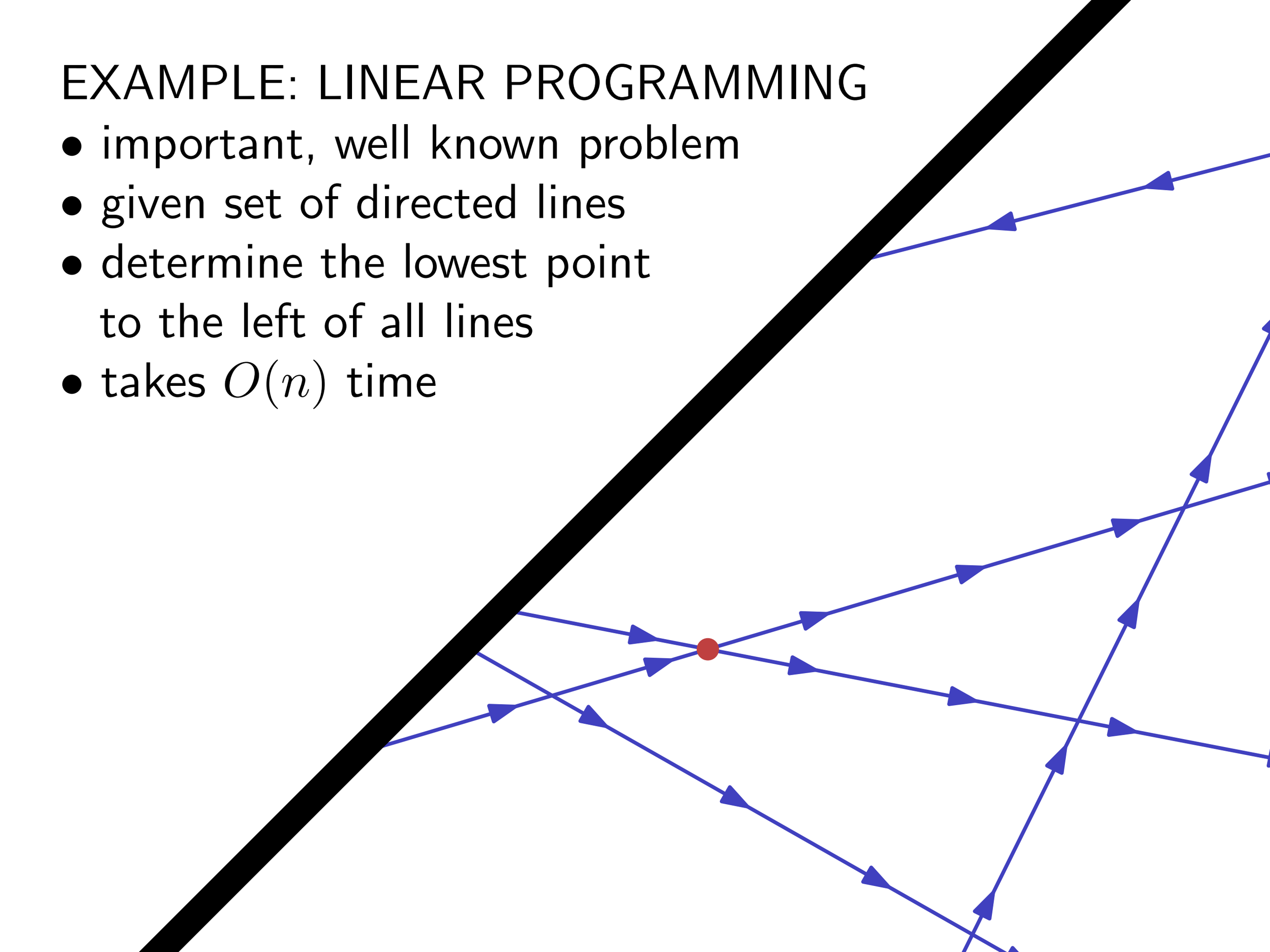
EXAMPLE: LINEAR PROGRAMMING

- important, well known problem
- given set of directed lines
- determine the lowest point
  to the left of all lines



EXAMPLE: LINEAR PROGRAMMING

- important, well known problem
- given set of directed lines
- determine the lowest point
  to the left of all lines
- takes $O(n)$ time



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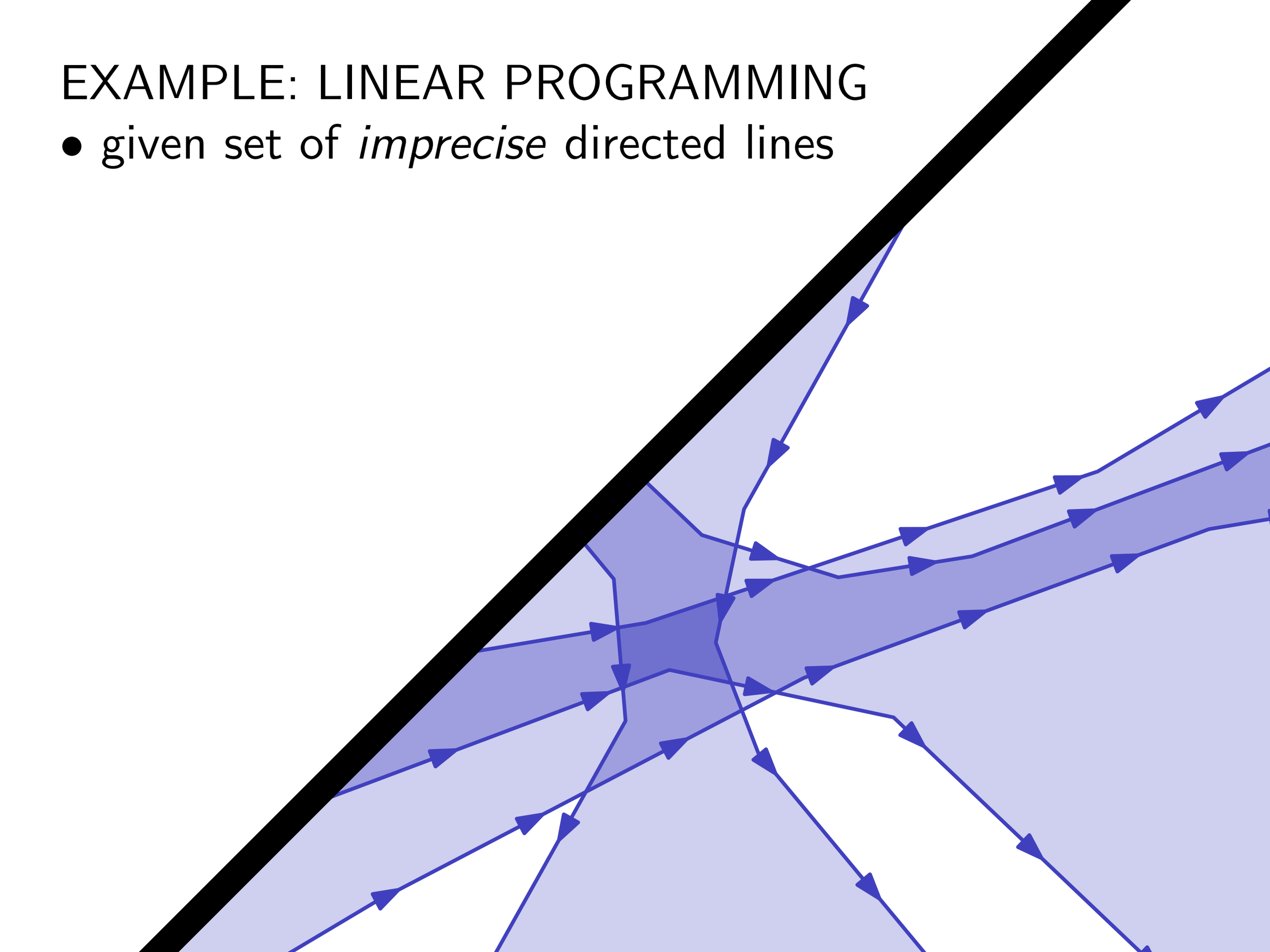
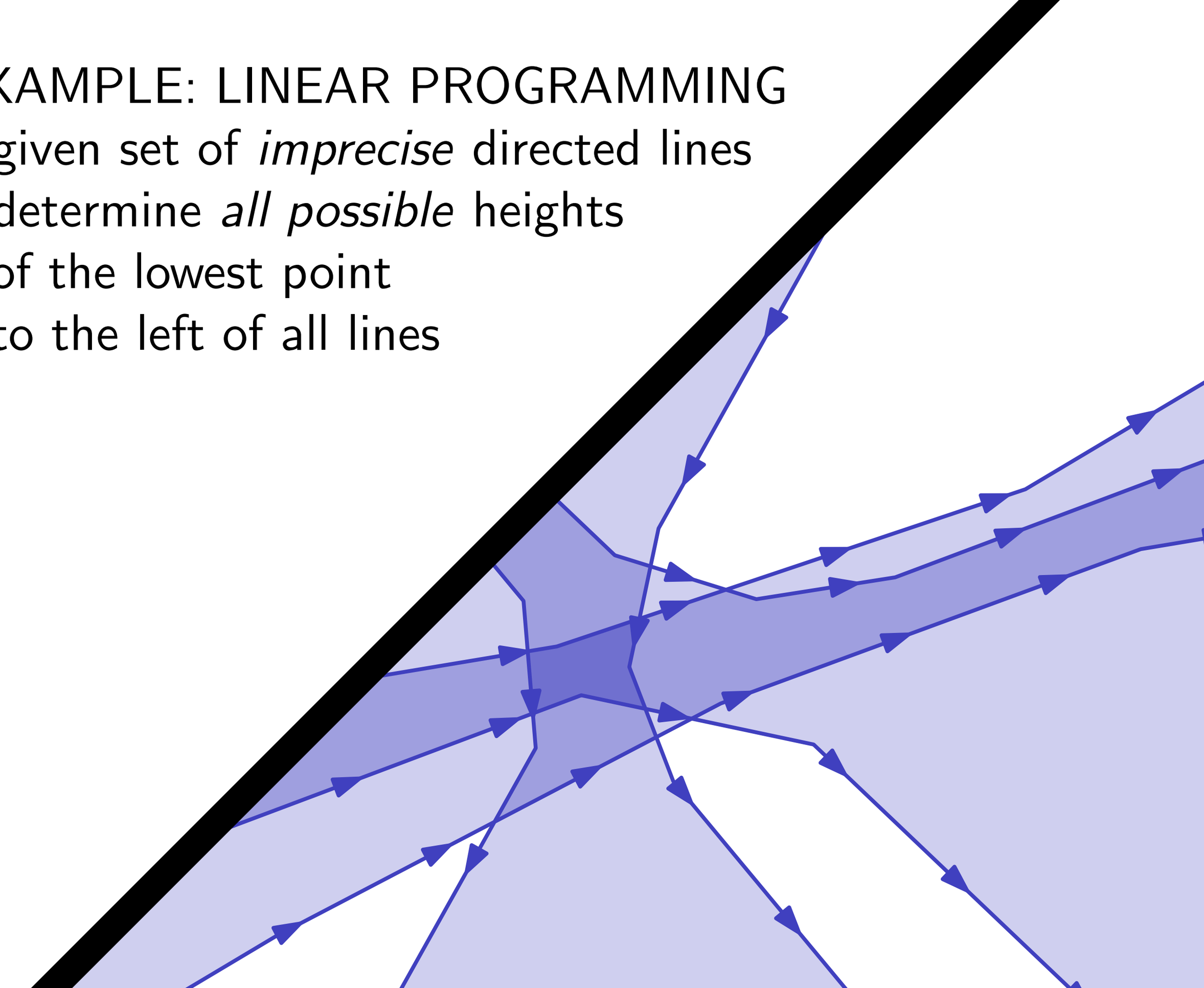
- given set of *imprecise* directed lines



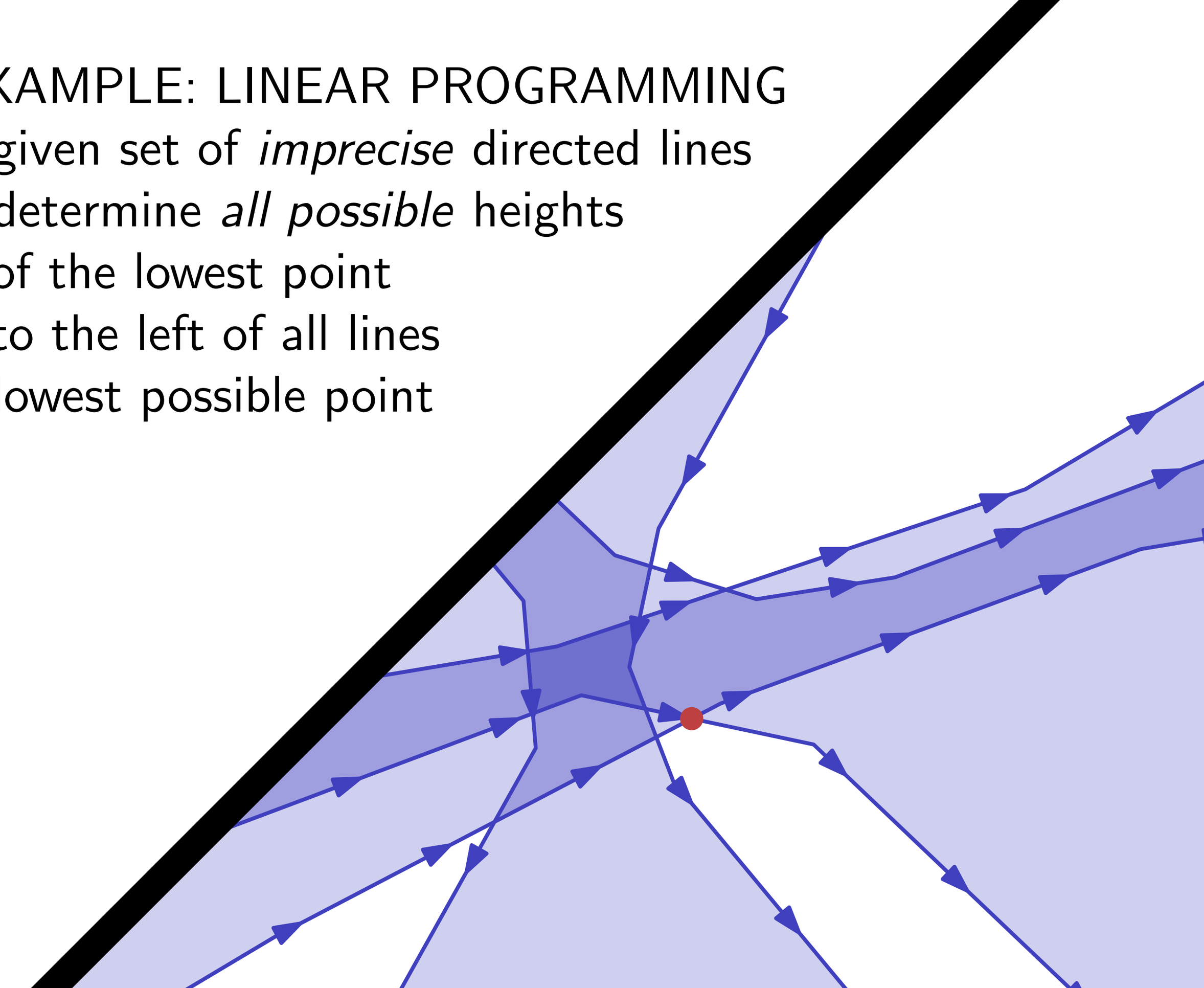
EXAMPLE: LINEAR PROGRAMMING

- given set of *imprecise* directed lines
- determine *all possible* heights
  of the lowest point
  to the left of all lines



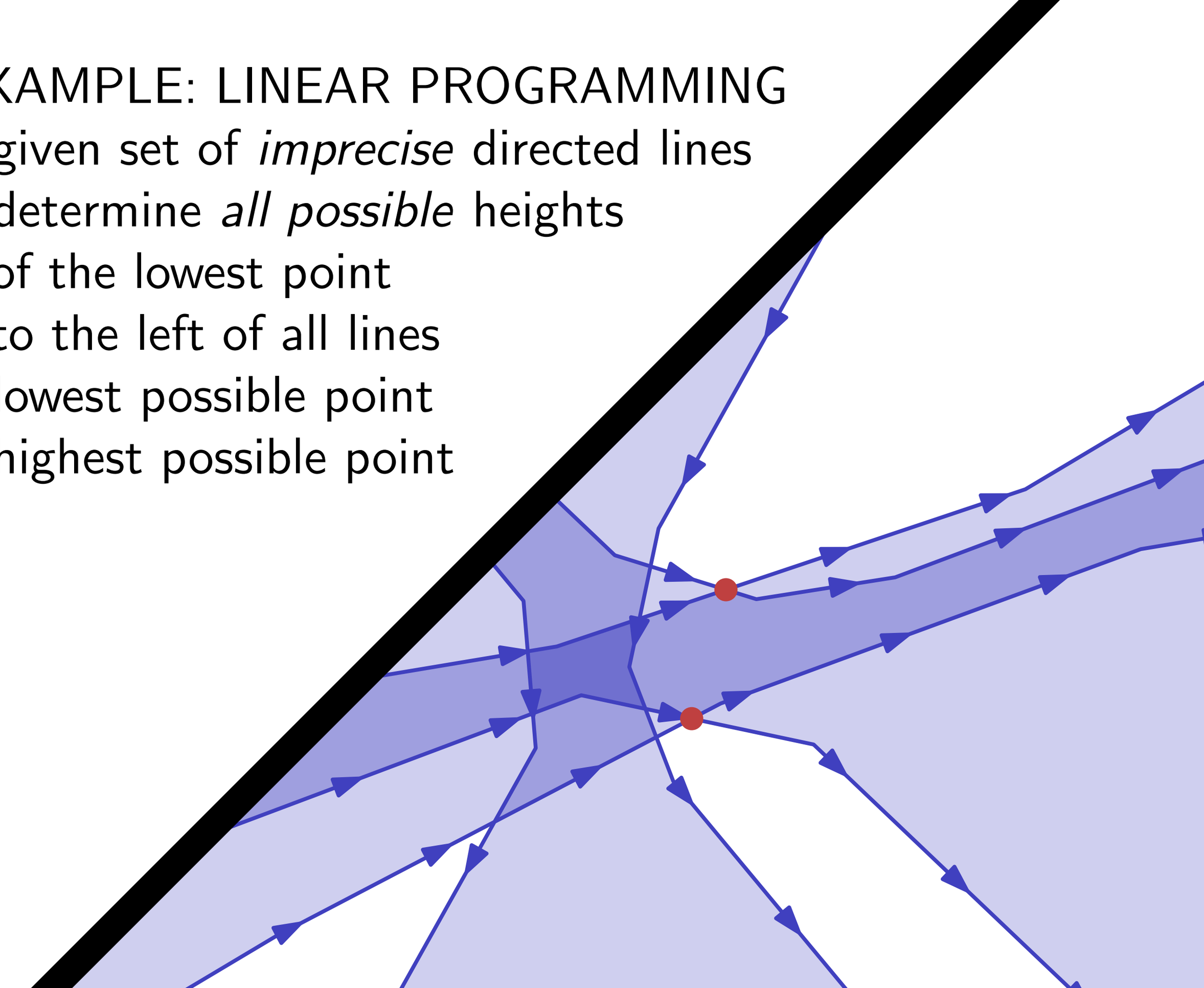
EXAMPLE: LINEAR PROGRAMMING

- given set of *imprecise* directed lines
- determine *all possible* heights
  of the lowest point
  to the left of all lines
- lowest possible point

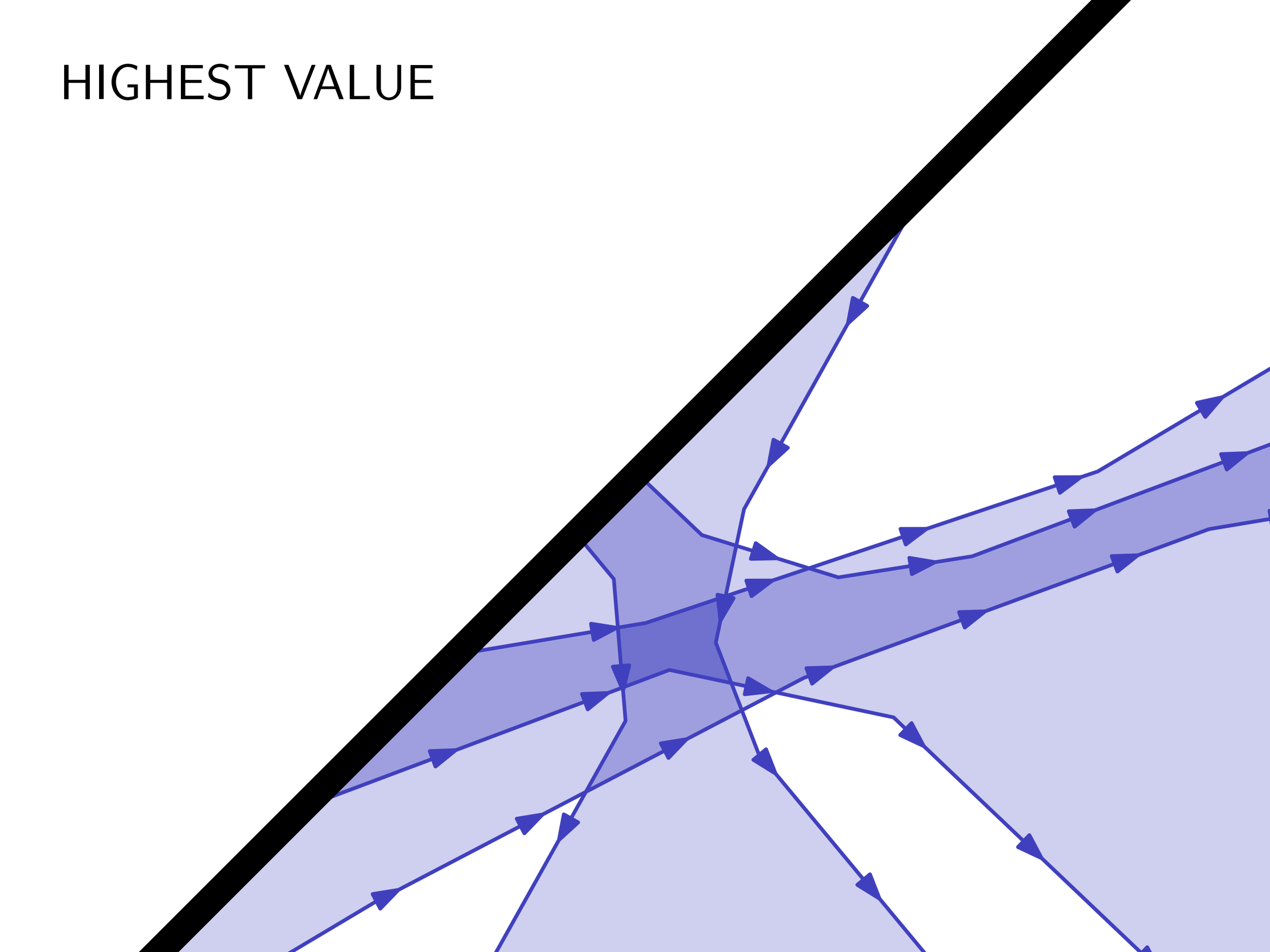


EXAMPLE: LINEAR PROGRAMMING

- given set of *imprecise* directed lines
- determine *all possible* heights
  of the lowest point
  to the left of all lines
- lowest possible point
- highest possible point

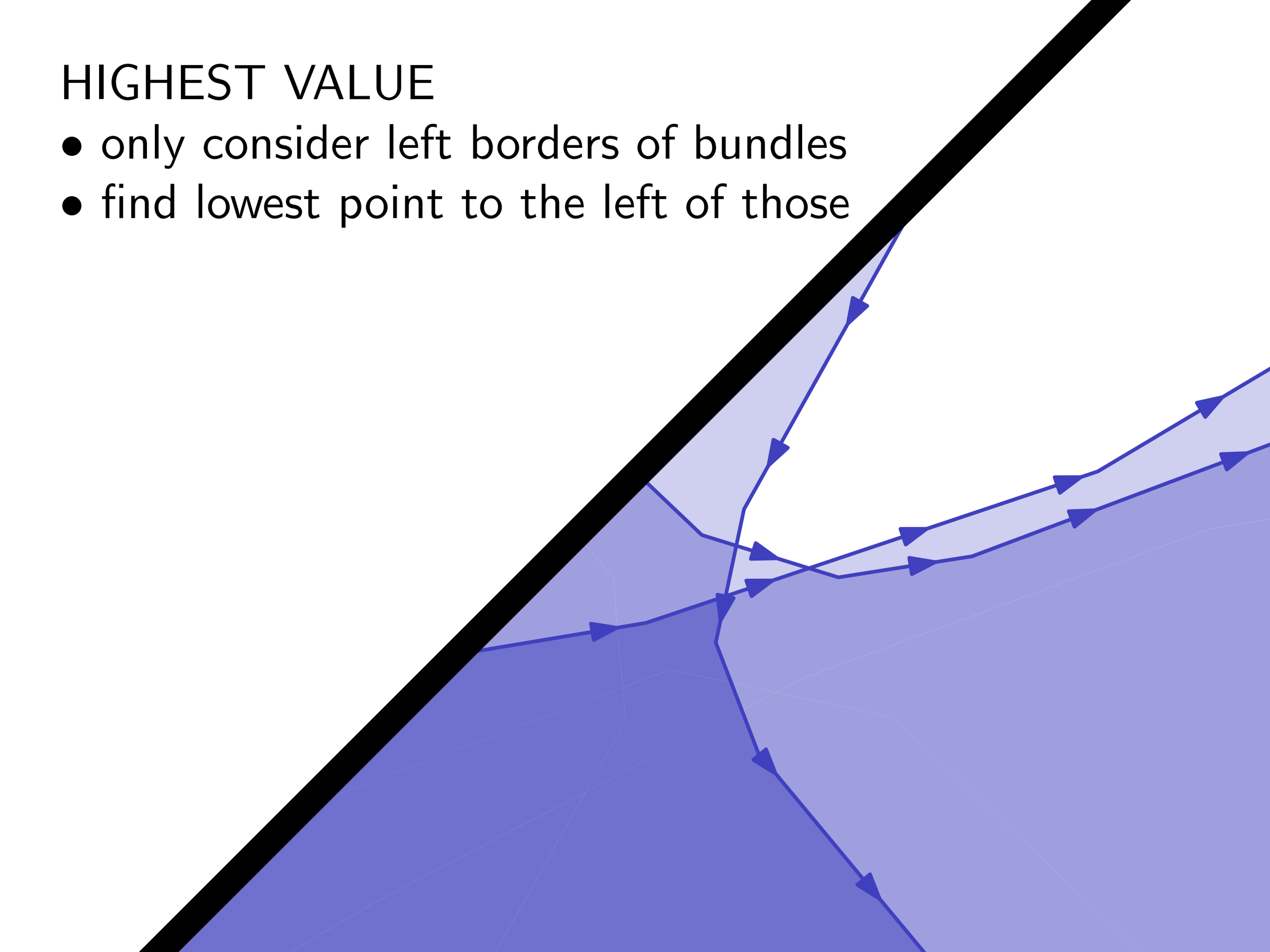


HIGHEST VALUE



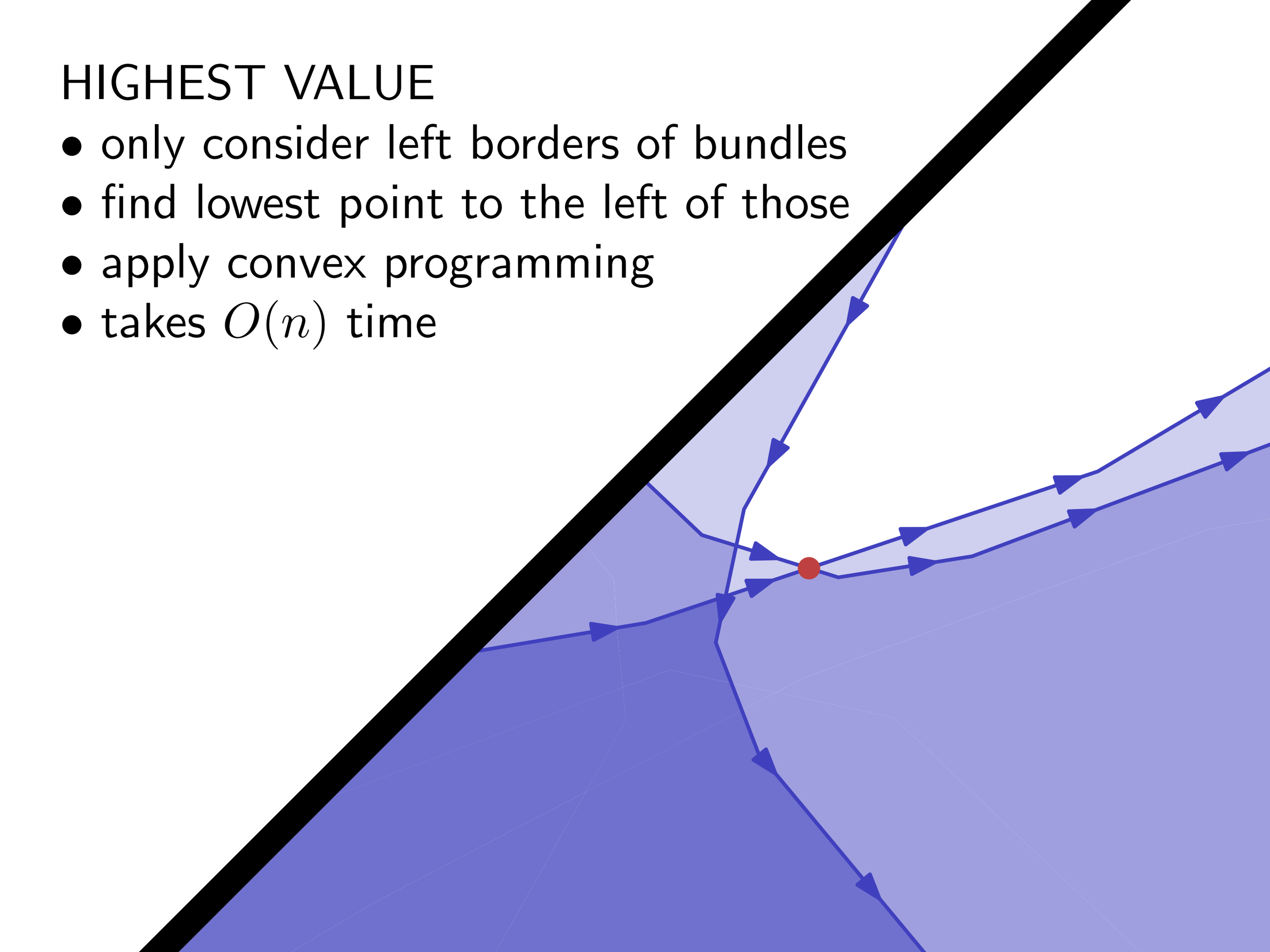
HIGHEST VALUE

- only consider left borders of bundles
- find lowest point to the left of those

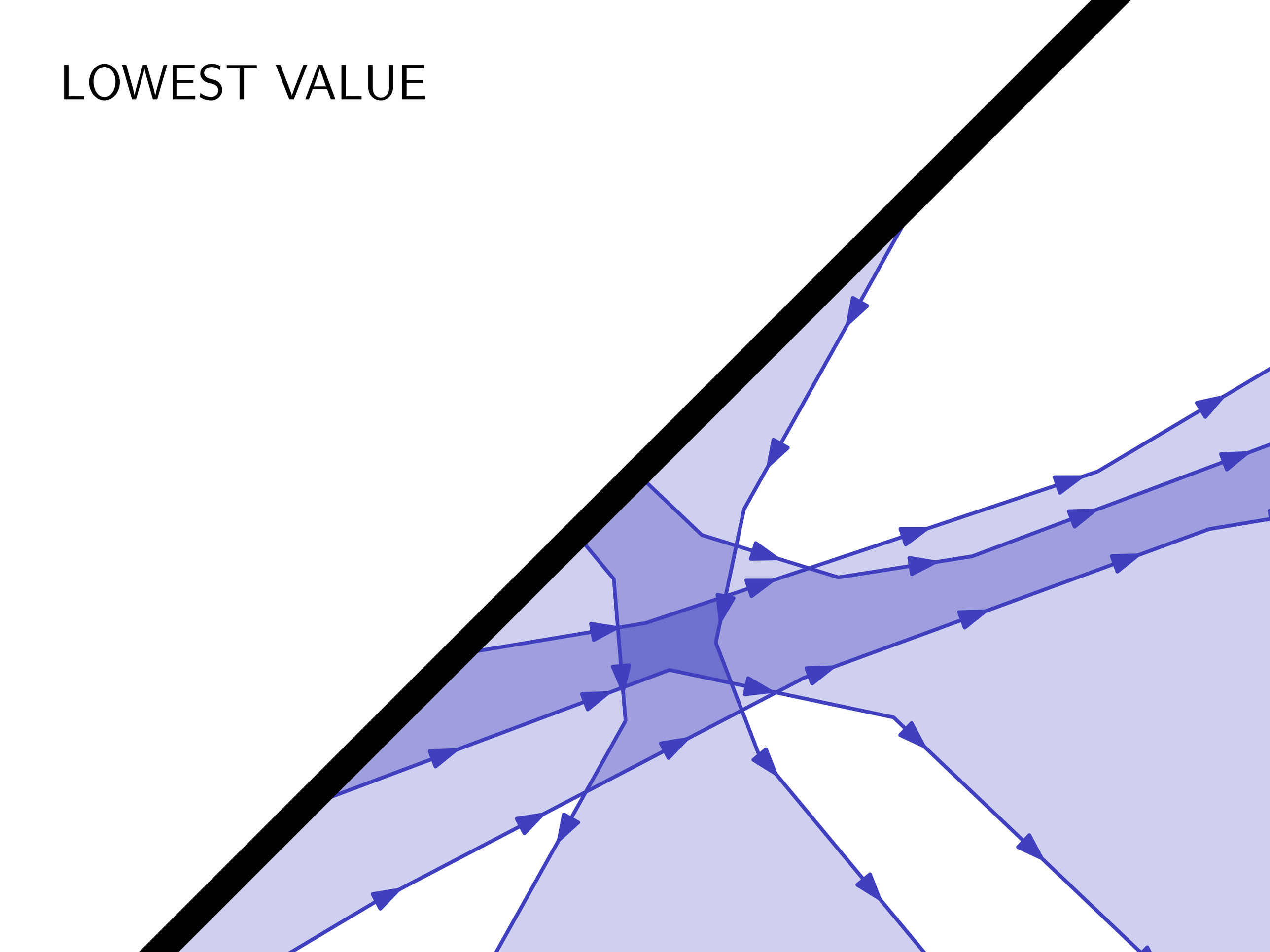


HIGHEST VALUE

- only consider left borders of bundles
- find lowest point to the left of those
- apply convex programming
- takes $O(n)$ time

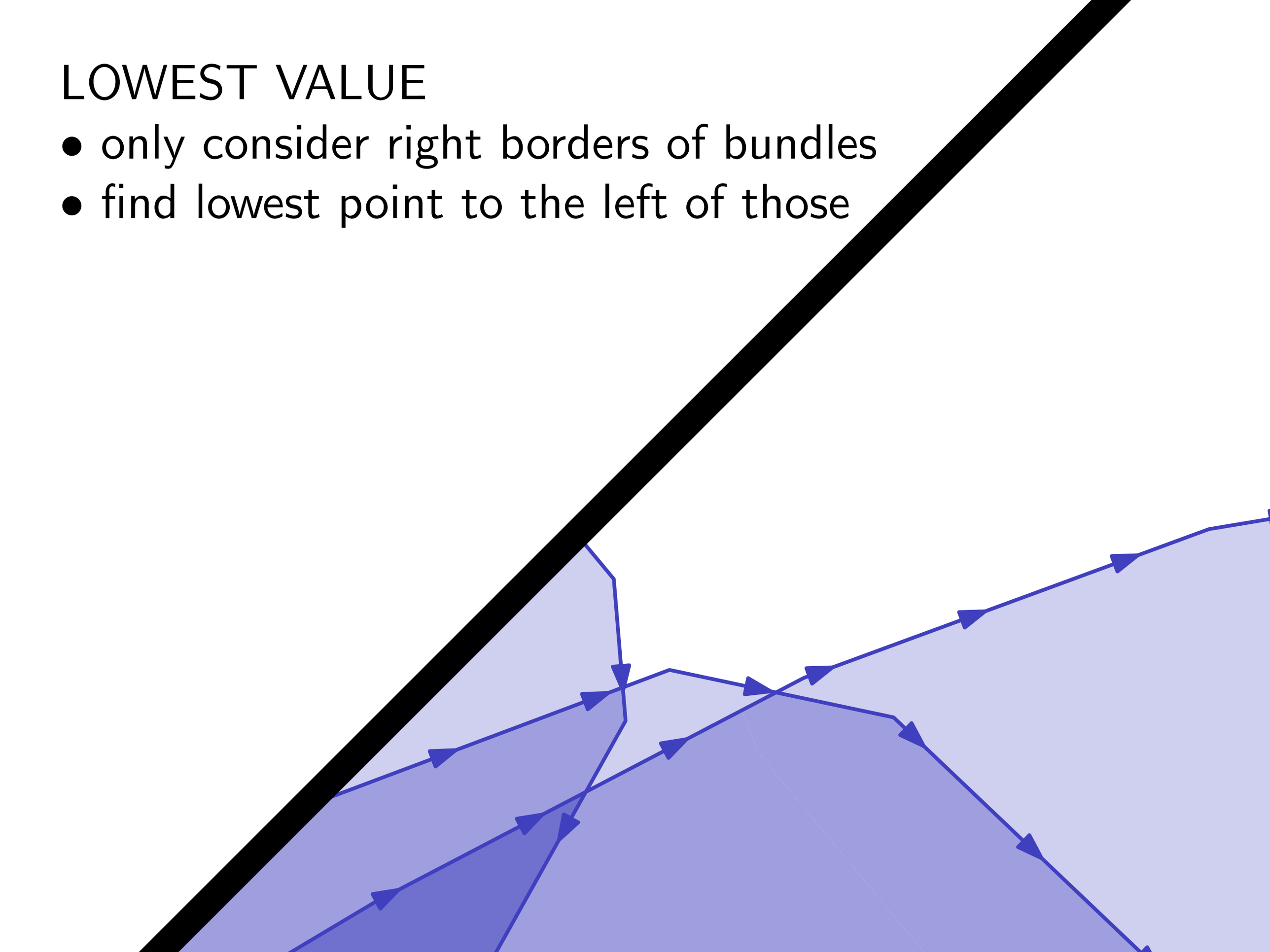


LOWEST VALUE



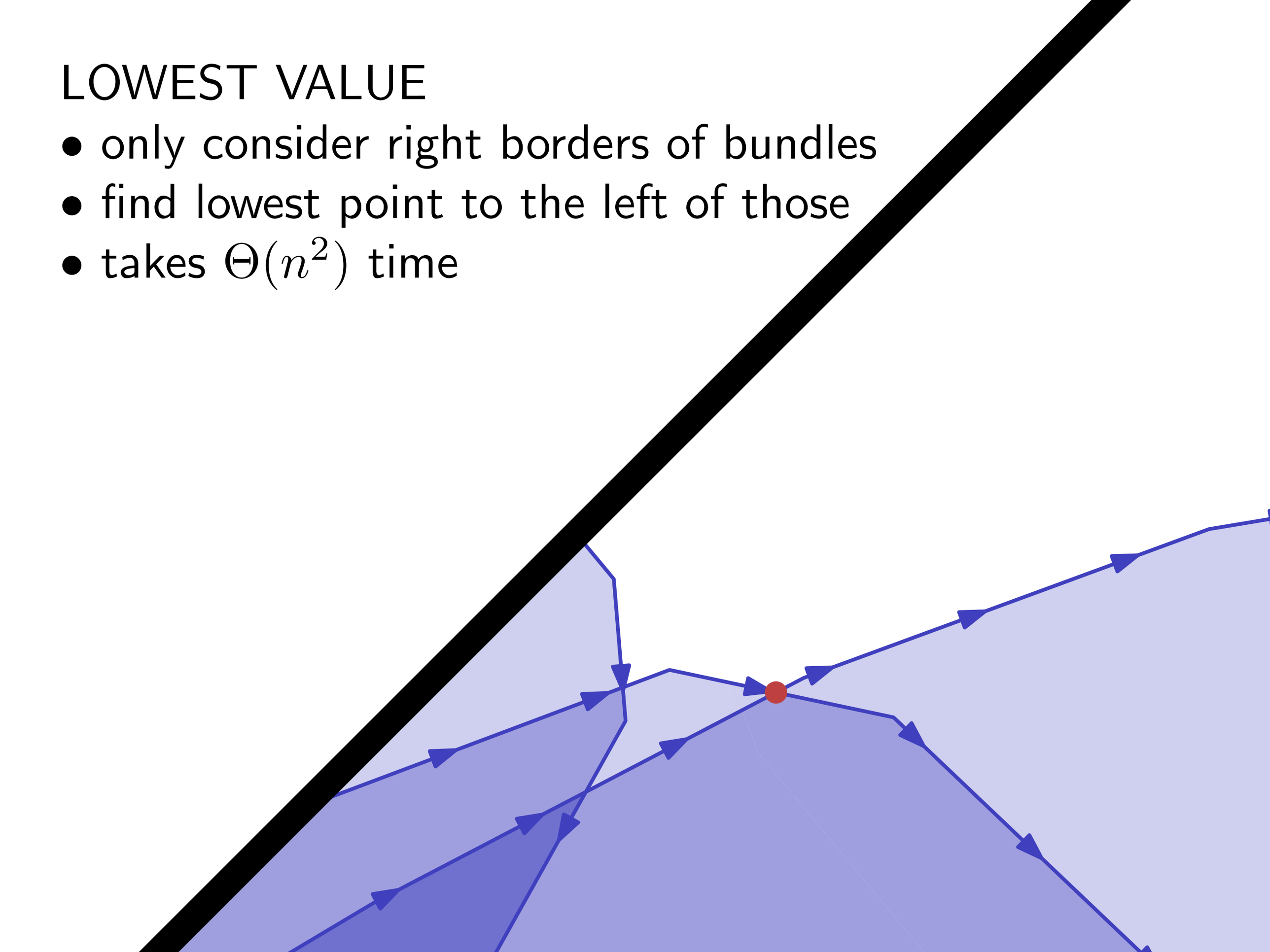
LOWEST VALUE

- only consider right borders of bundles
- find lowest point to the left of those



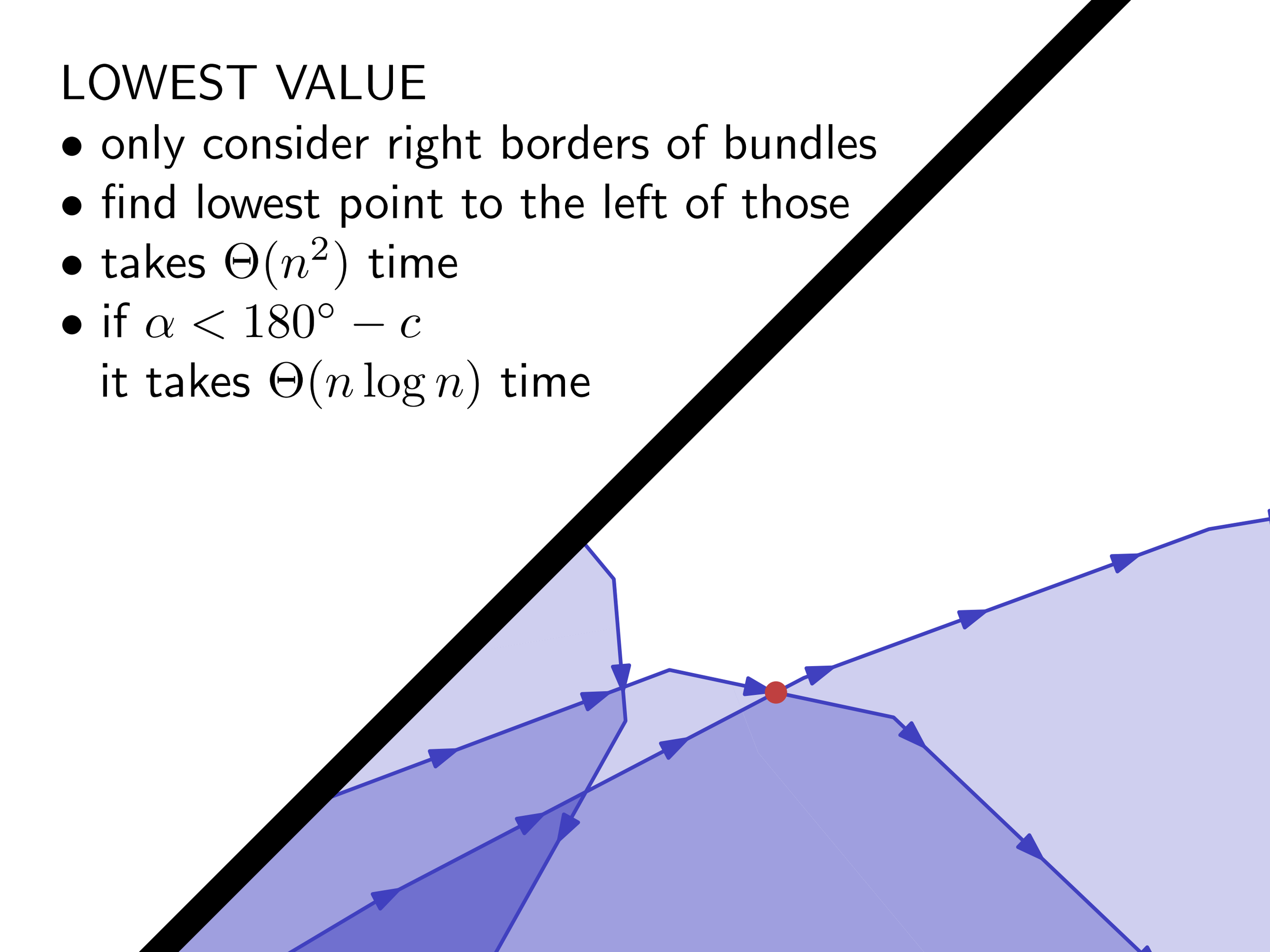
LOWEST VALUE

- only consider right borders of bundles
- find lowest point to the left of those
- takes $\Theta(n^2)$ time



LOWEST VALUE

- only consider right borders of bundles
- find lowest point to the left of those
- takes $\Theta(n^2)$ time
- if $\alpha < 180° - c$
  it takes $\Theta(n \log n)$ time



Thank You!

Questions?