I/O Efficient Sorting in the Cache-Oblivious Model: Quicker than Quick

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Carleton University

Overview

- Previous Work
- Introduce Cache-Oblivious Model
- Sorting Algorithm: *Funnelsort*
- Funnelsort in multi-core environment

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- Cache-Oblivious Algorithms
  - Introduced in 1999 – Frigo, Leiserson, Prokop, Ramachandran
  - **Cache Oblivious Model**
  - Funnelsort algorithm
- Improvements on Funnelsort
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  - should impart a *realistic* view of performance
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  - performed between CPU and main memory
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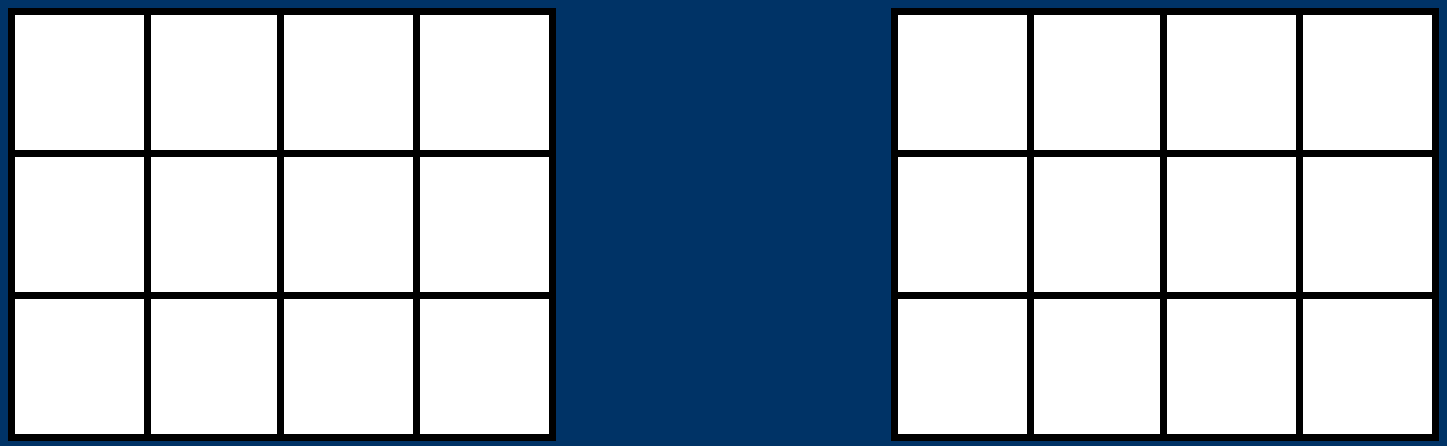

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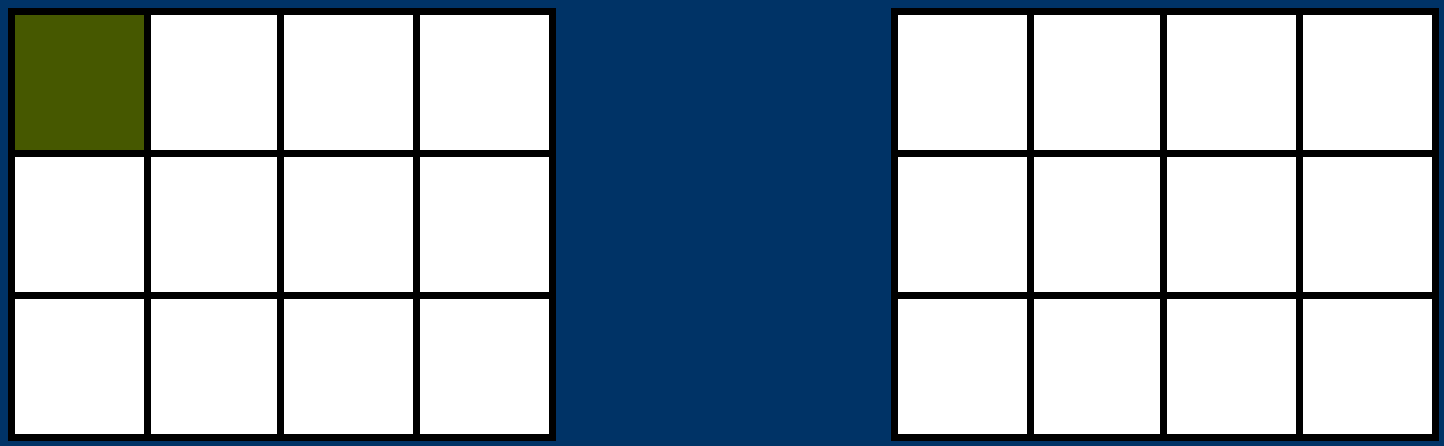

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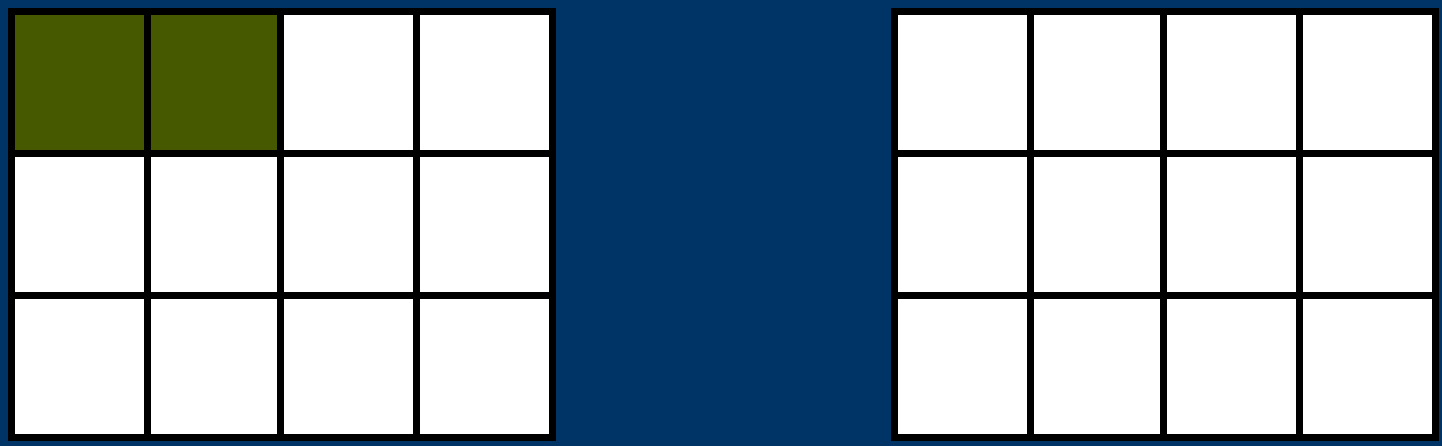

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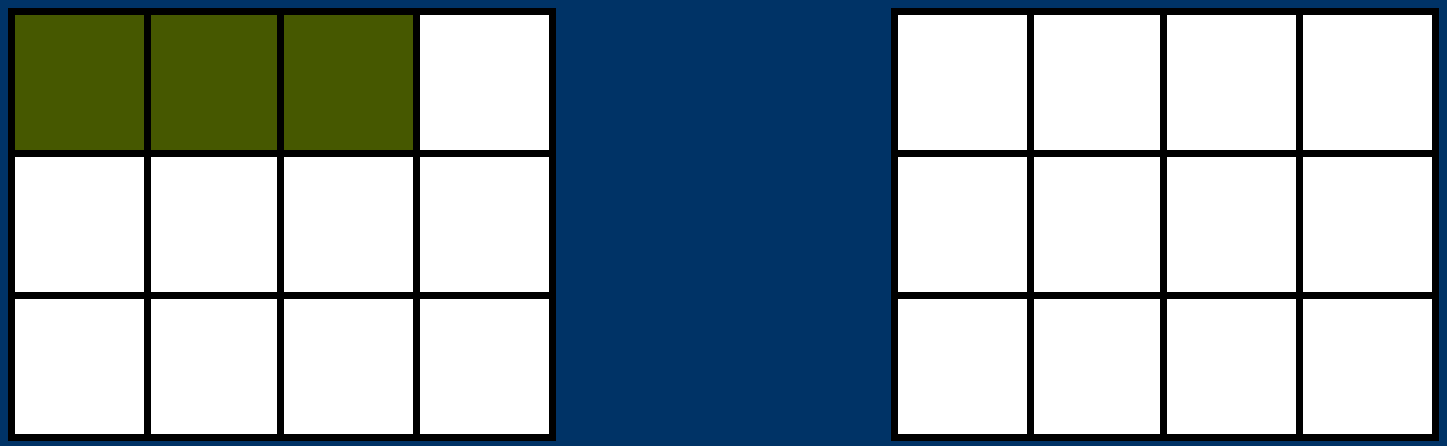

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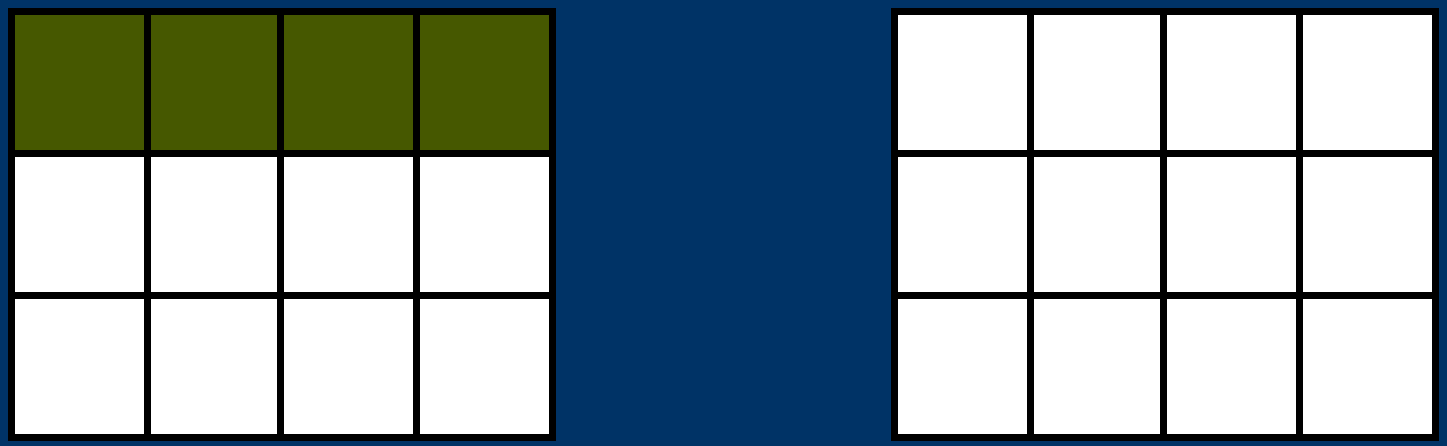

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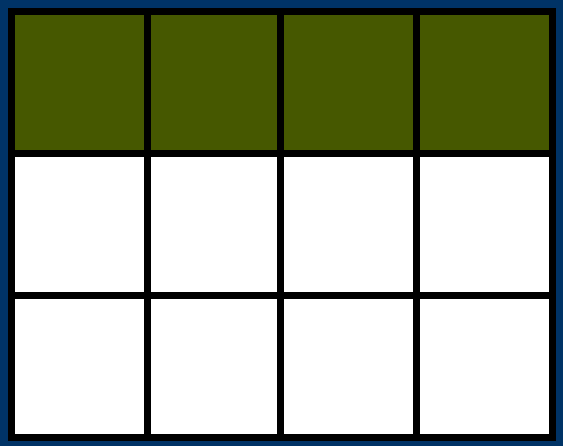

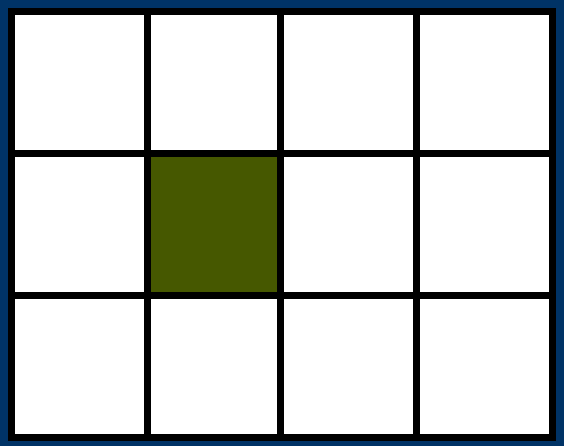

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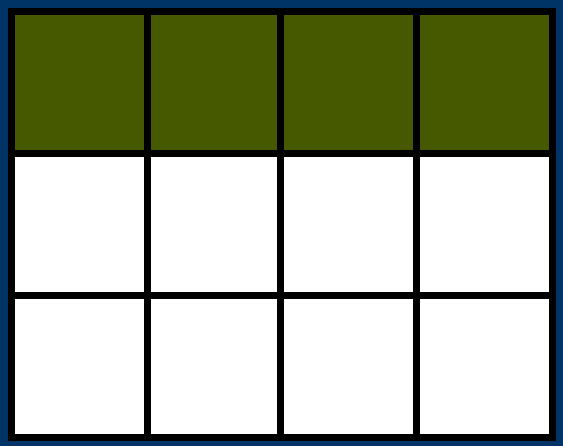

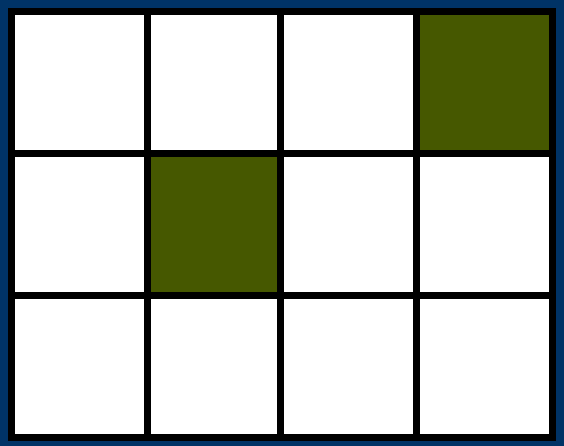

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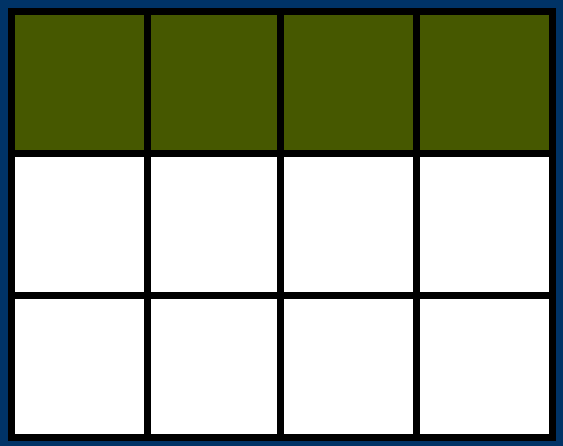

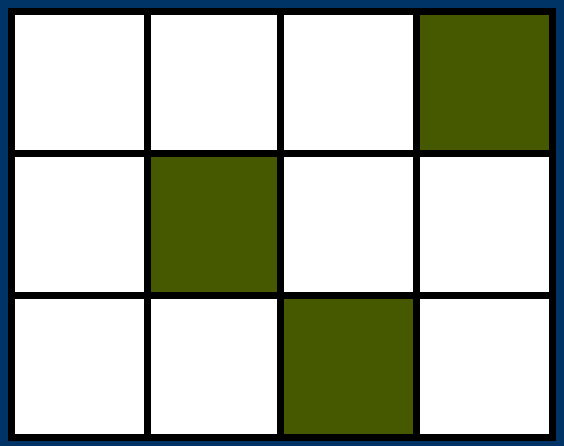

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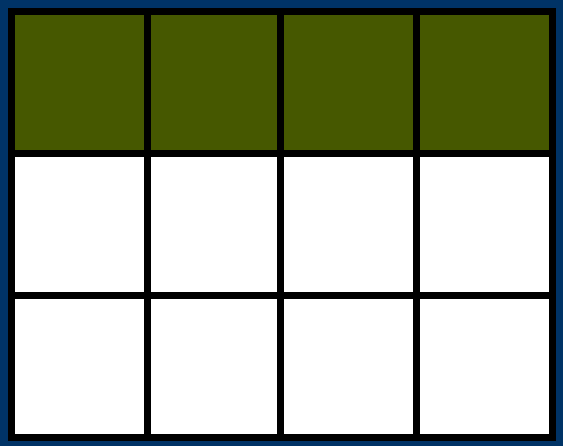

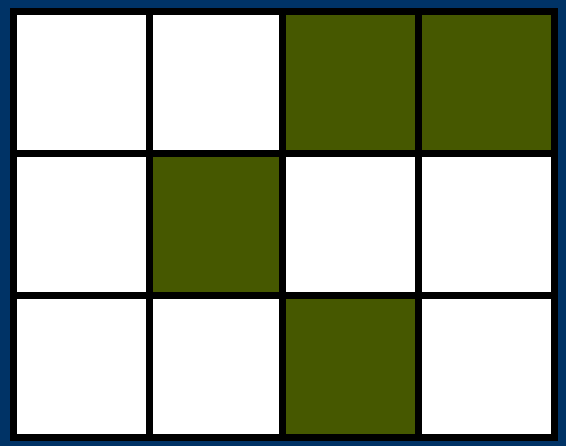

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- Want to measure the efficiency of an algorithm's memory use.

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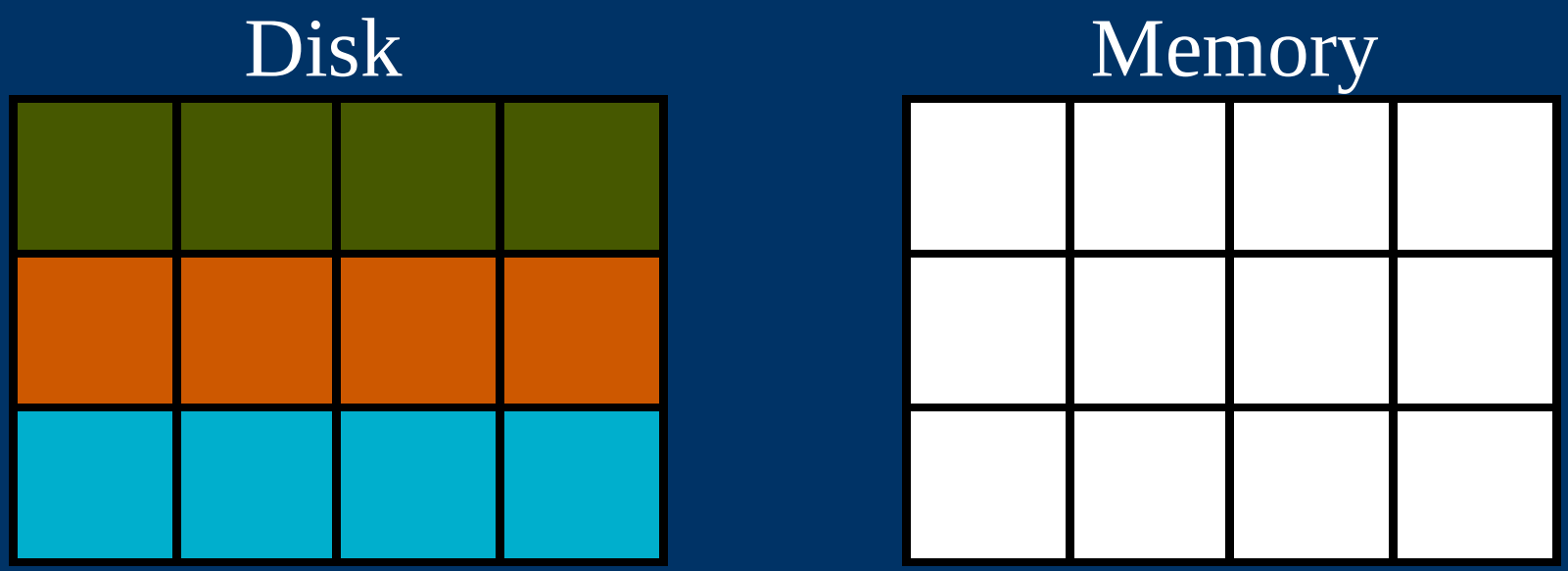

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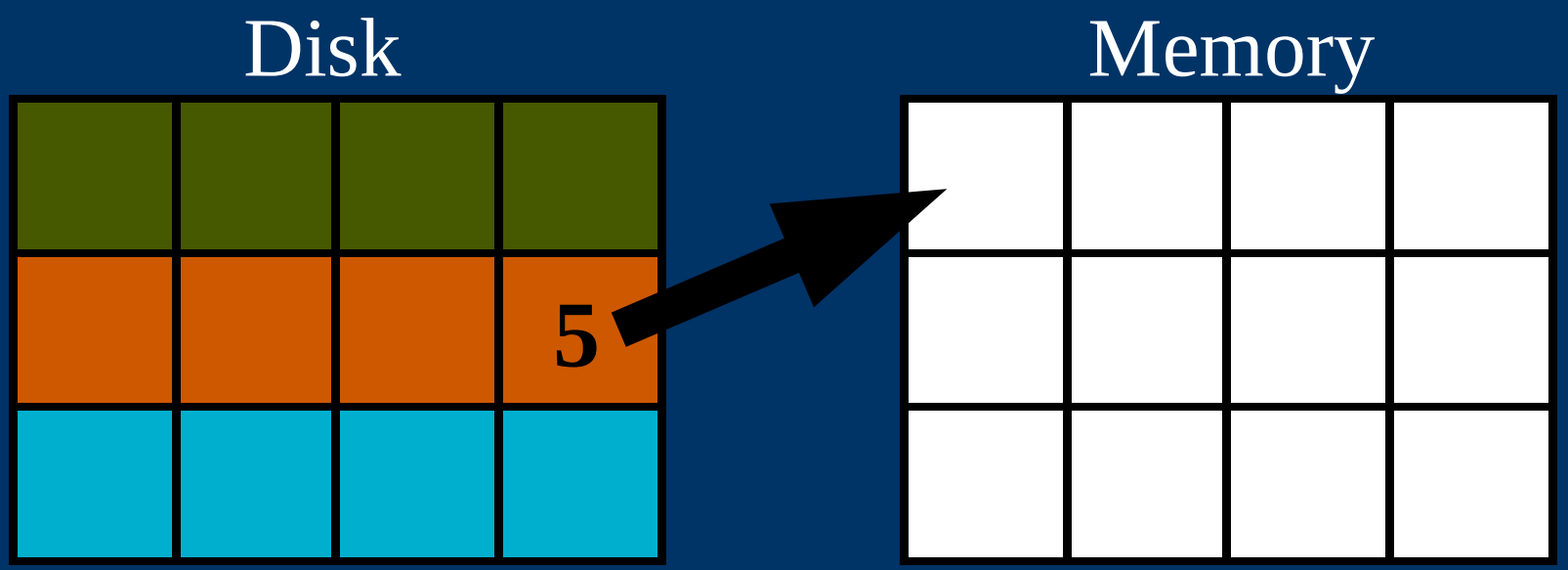

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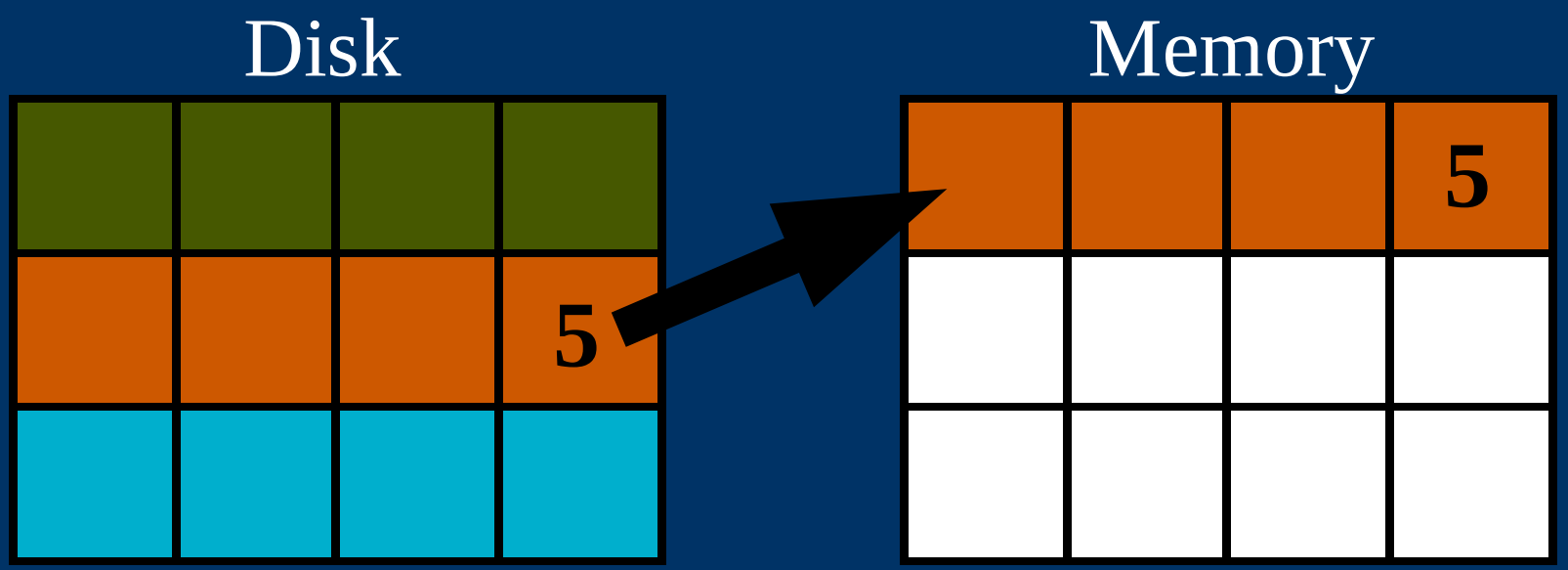

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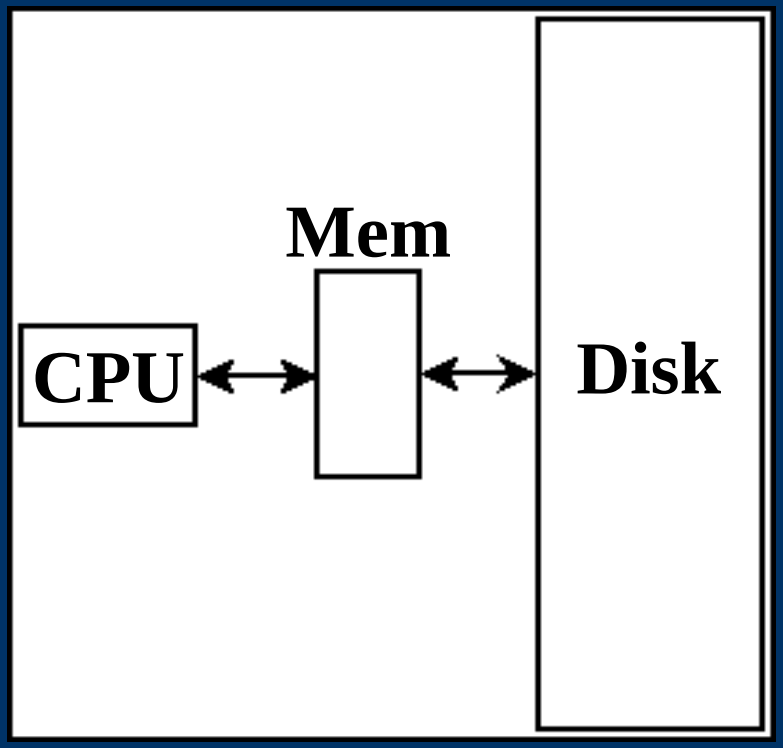

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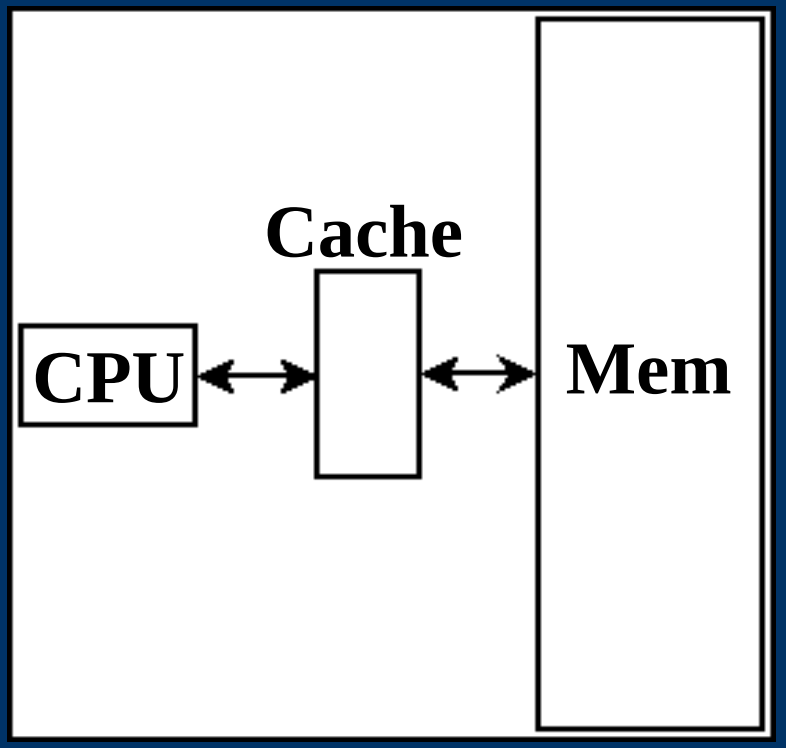

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- **Sorting algorithm**
- Similar to merge sort
- Both work and I/O optimal
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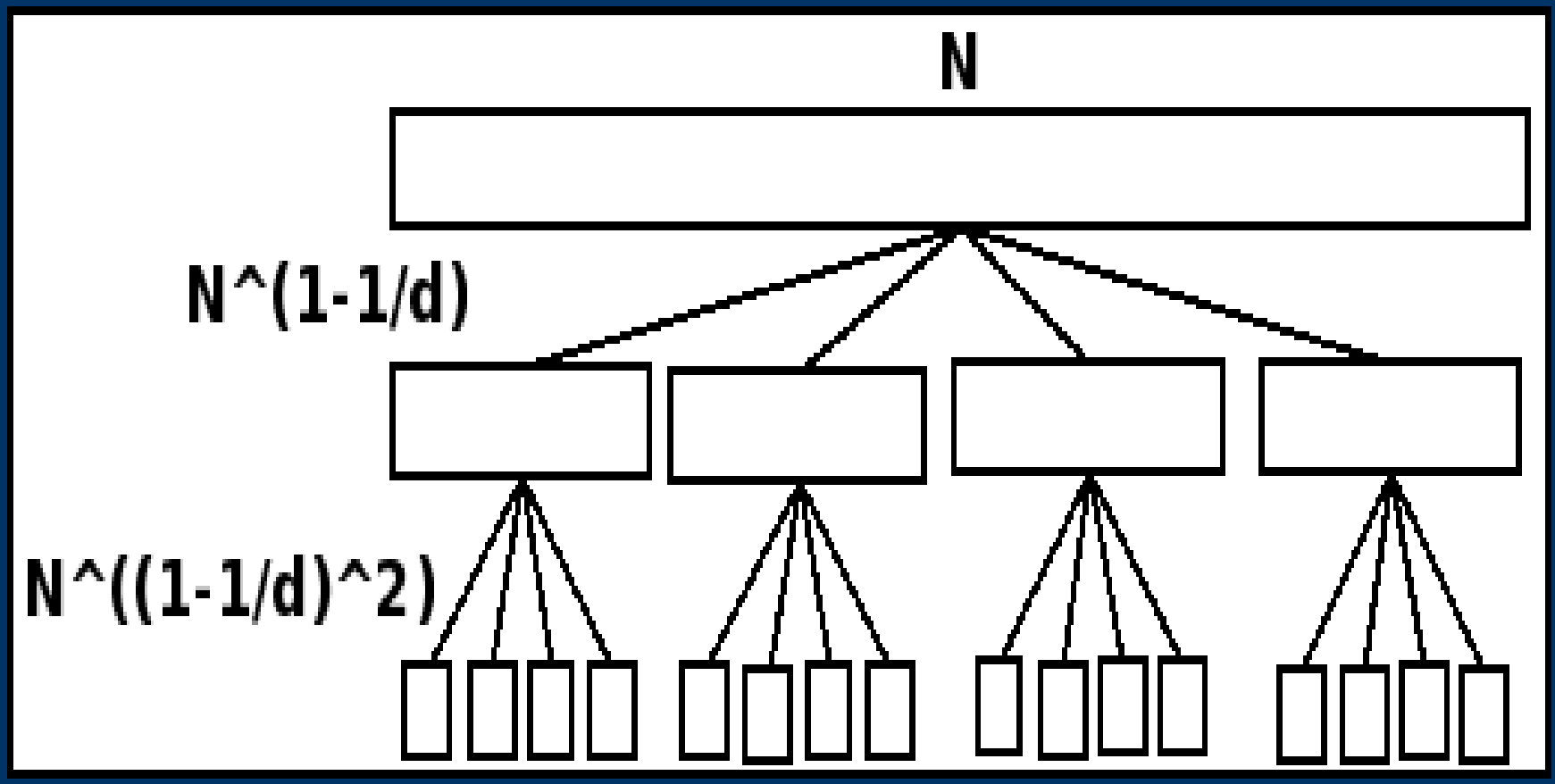
... Yes.

Funnelsort

- How does it work ?
  - Split the input into smaller groups
    - Split N elements into N^(1/d) groups of size N^(1-1/d)
  - Recursively sort each group
  - Merge the sorted groups together

Funnelsort

- Looks a lot like standard merge sort



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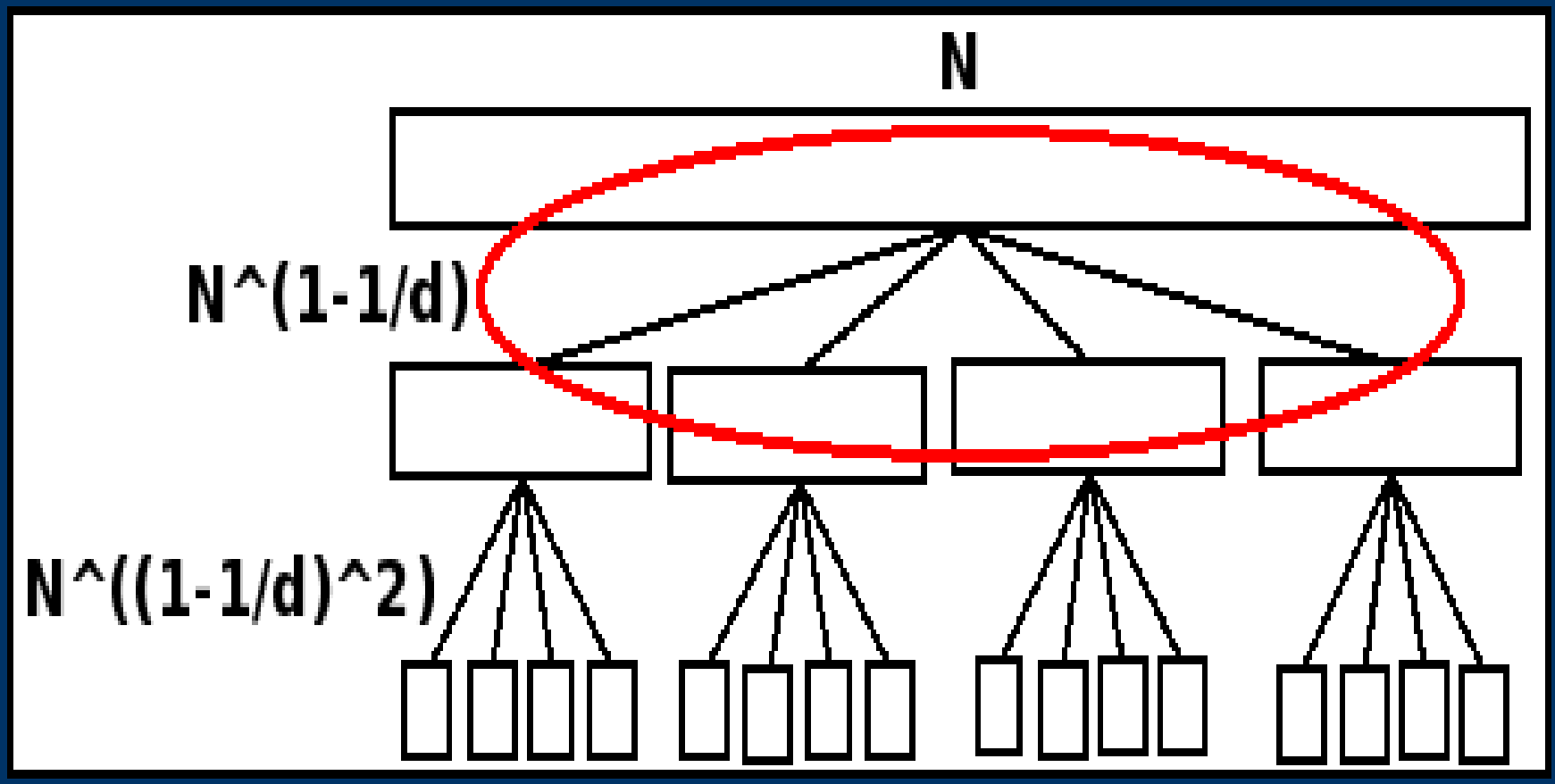
- Do merging in an I/O optimal way



Funnelsort - Merging Process

- **Merge together *k* groups of sorted input**
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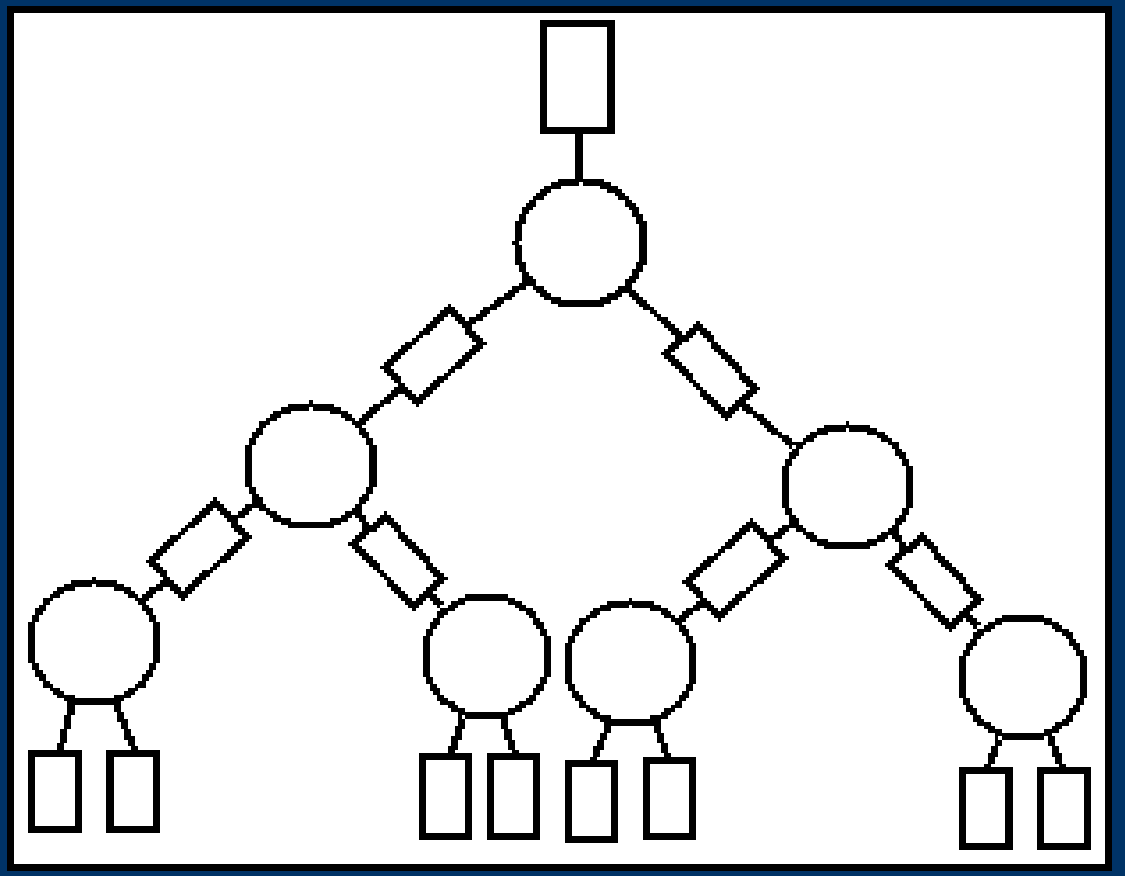
- k-merger (k=8)
- Boxes are buffers
- Each node merges into buffer above it



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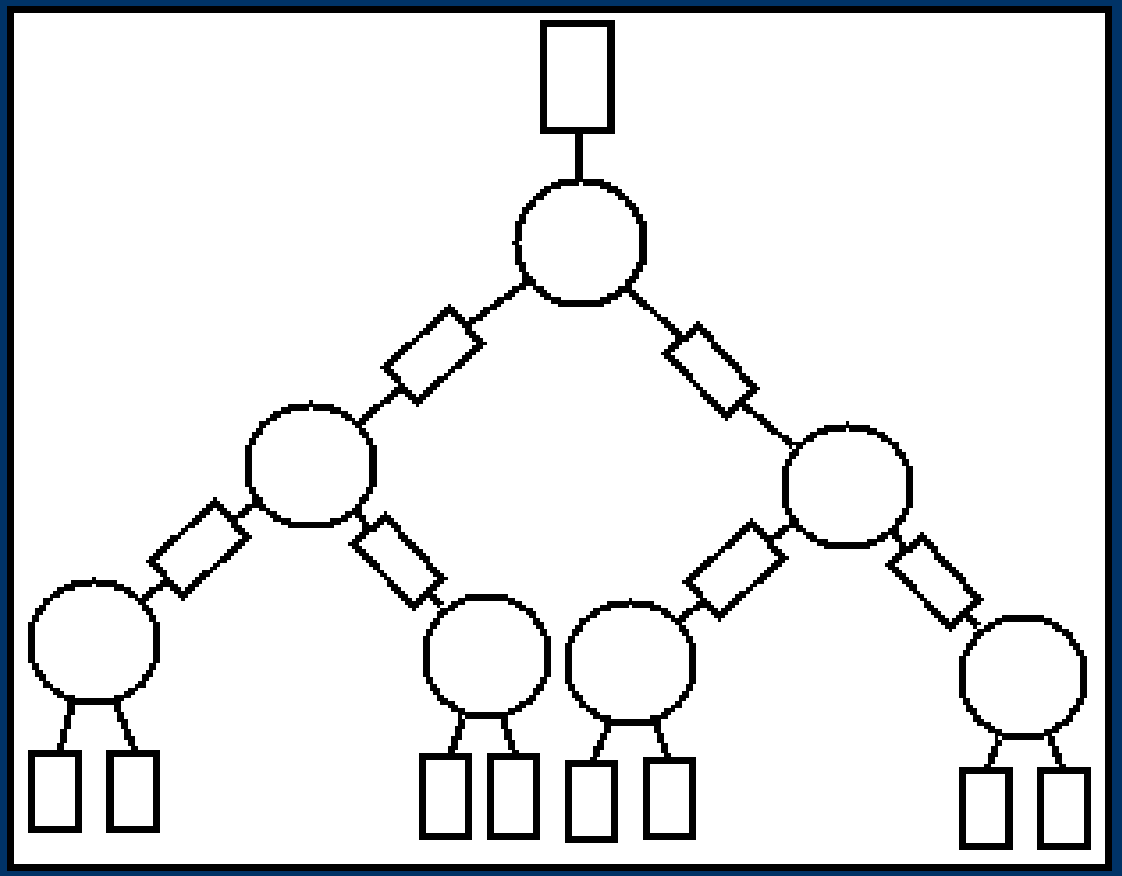

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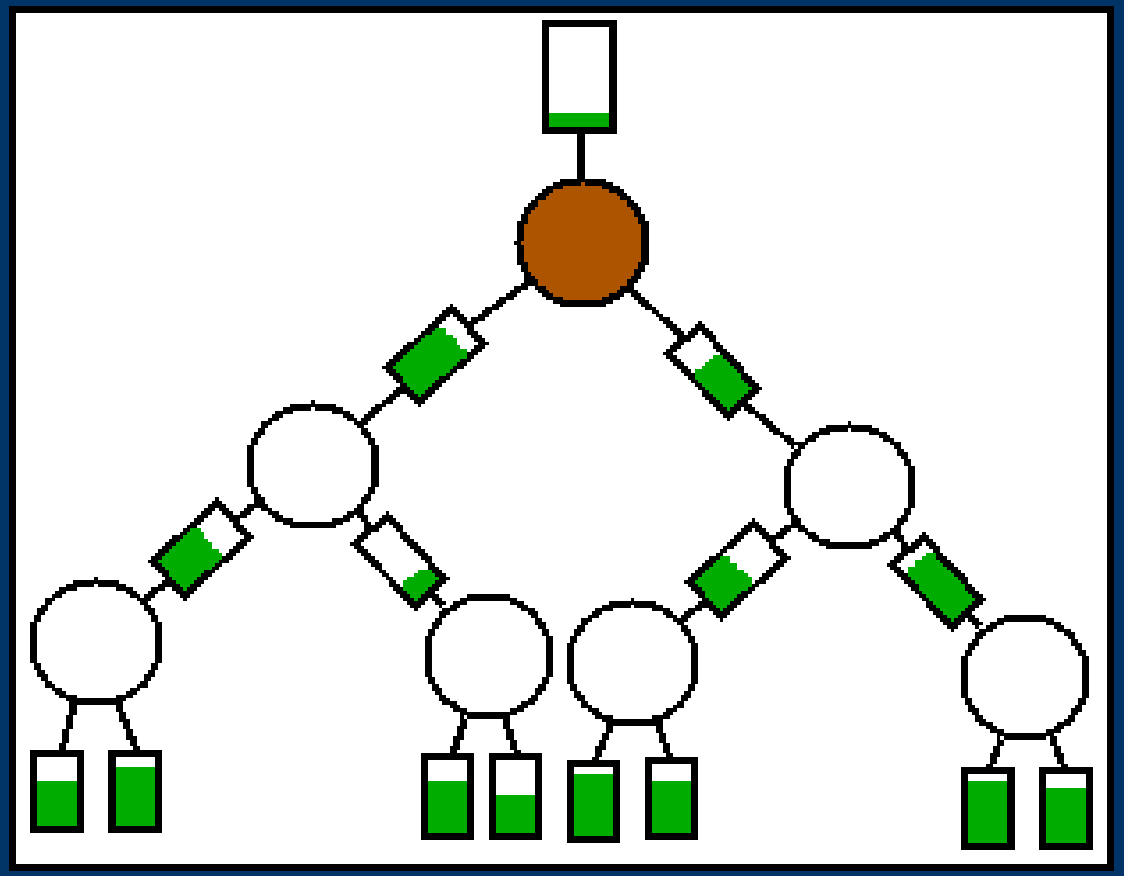

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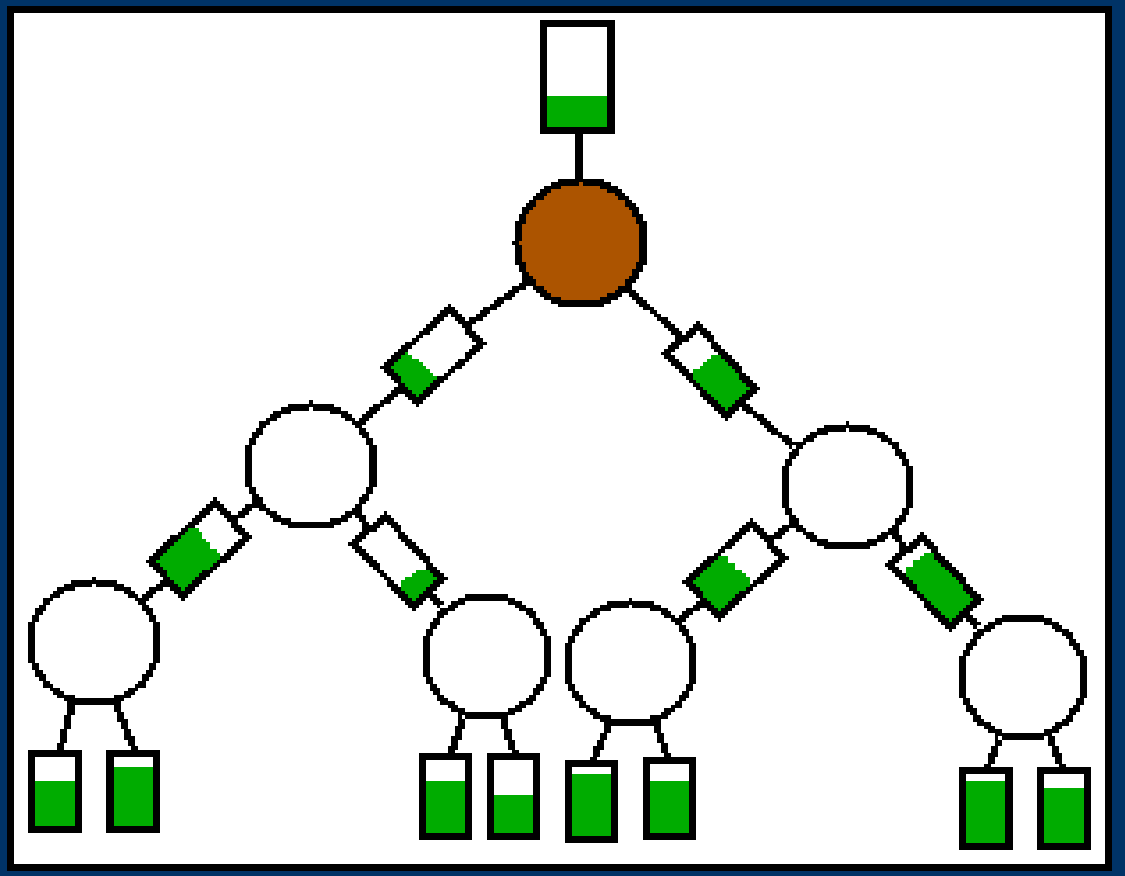

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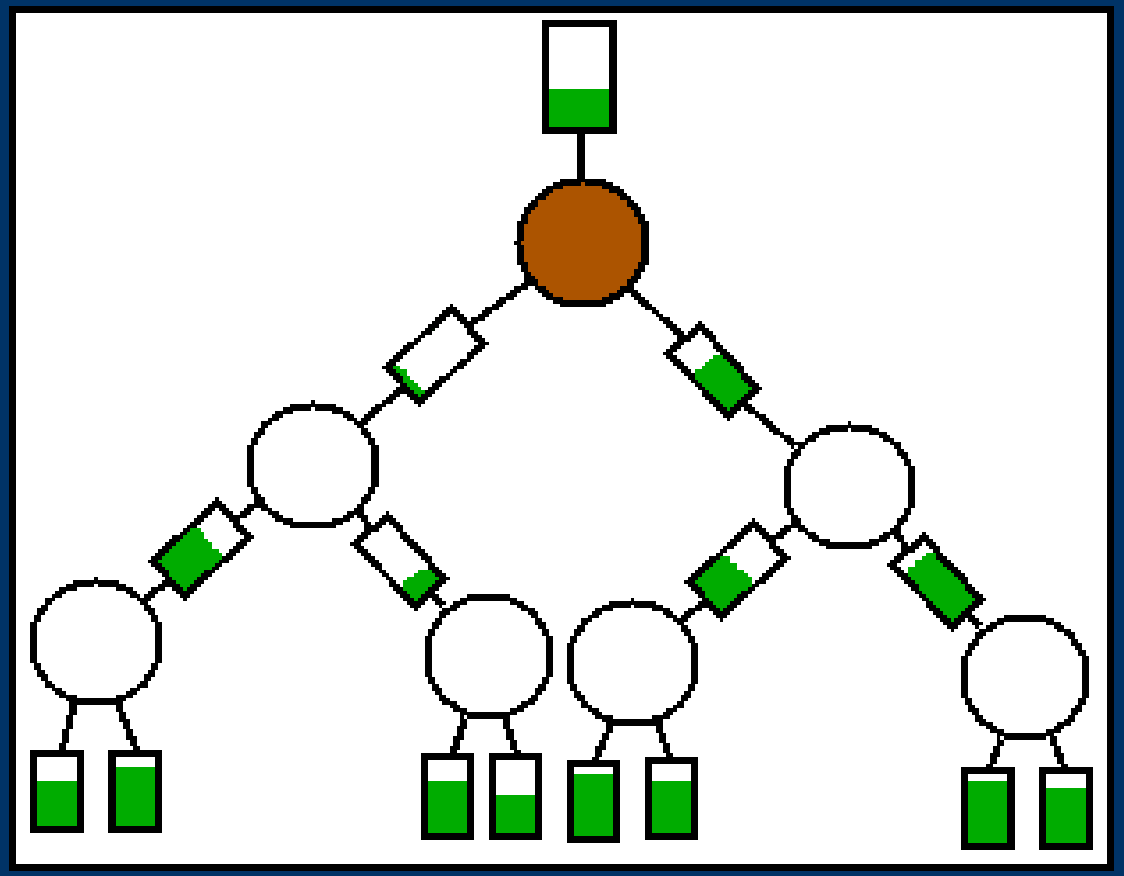

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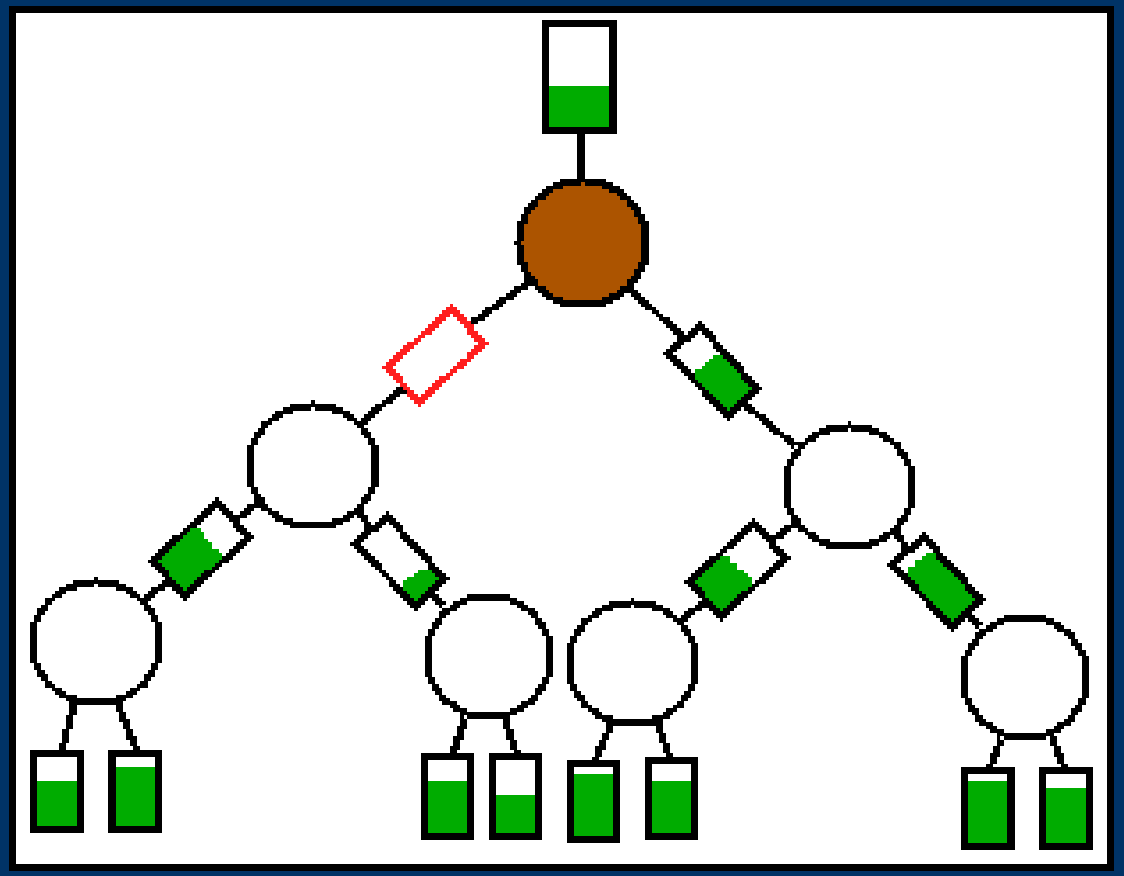

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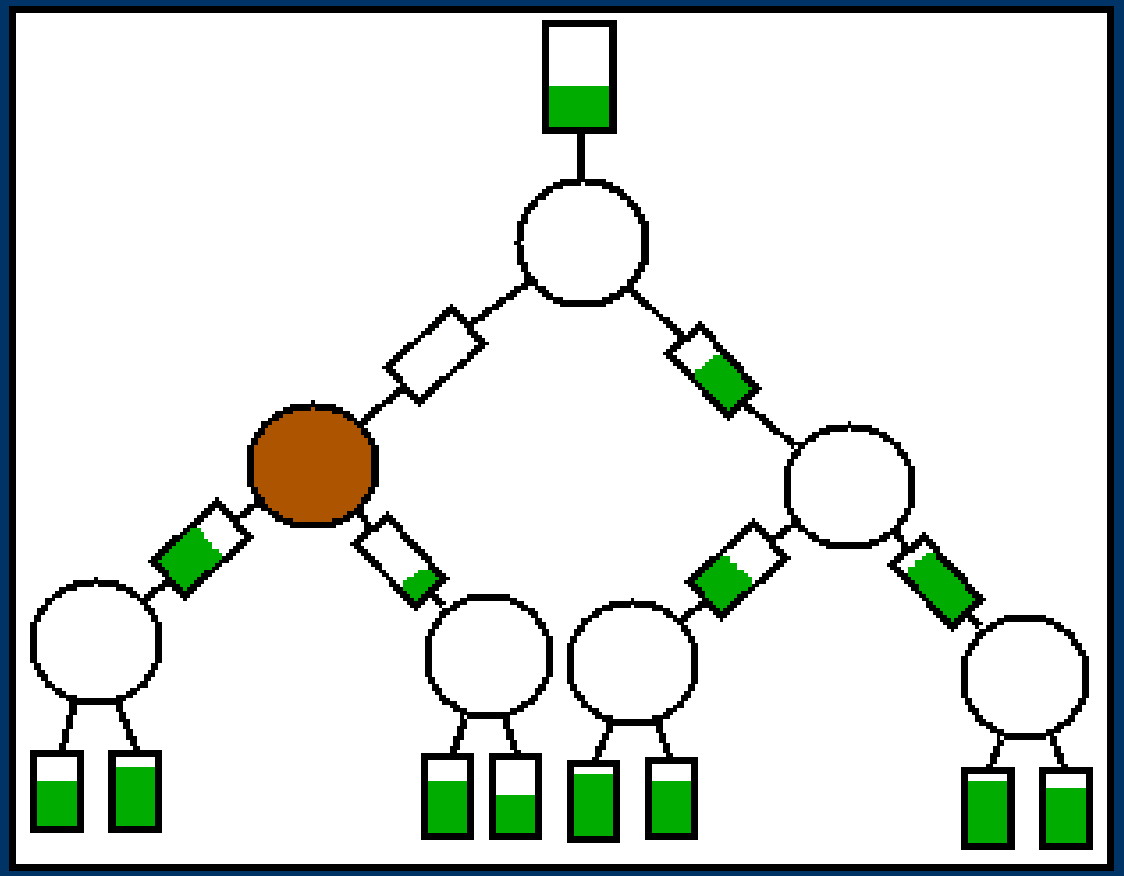

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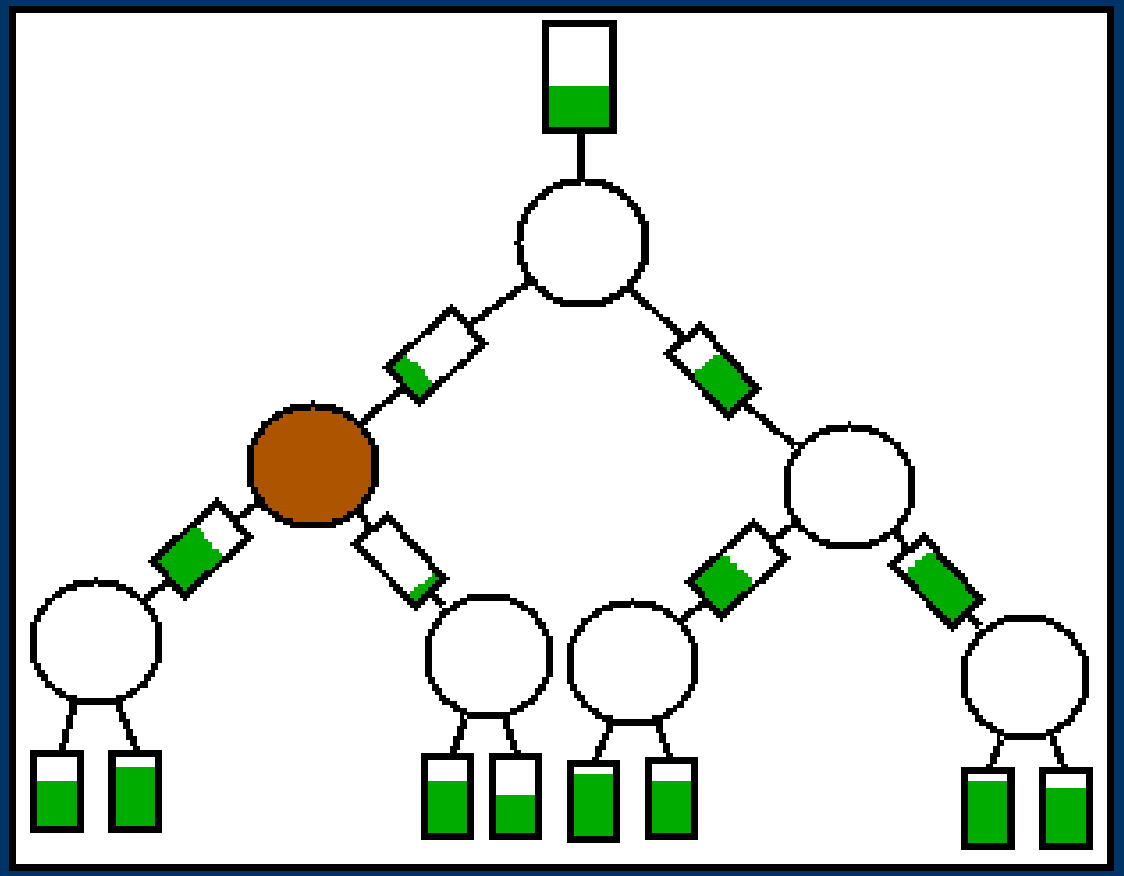

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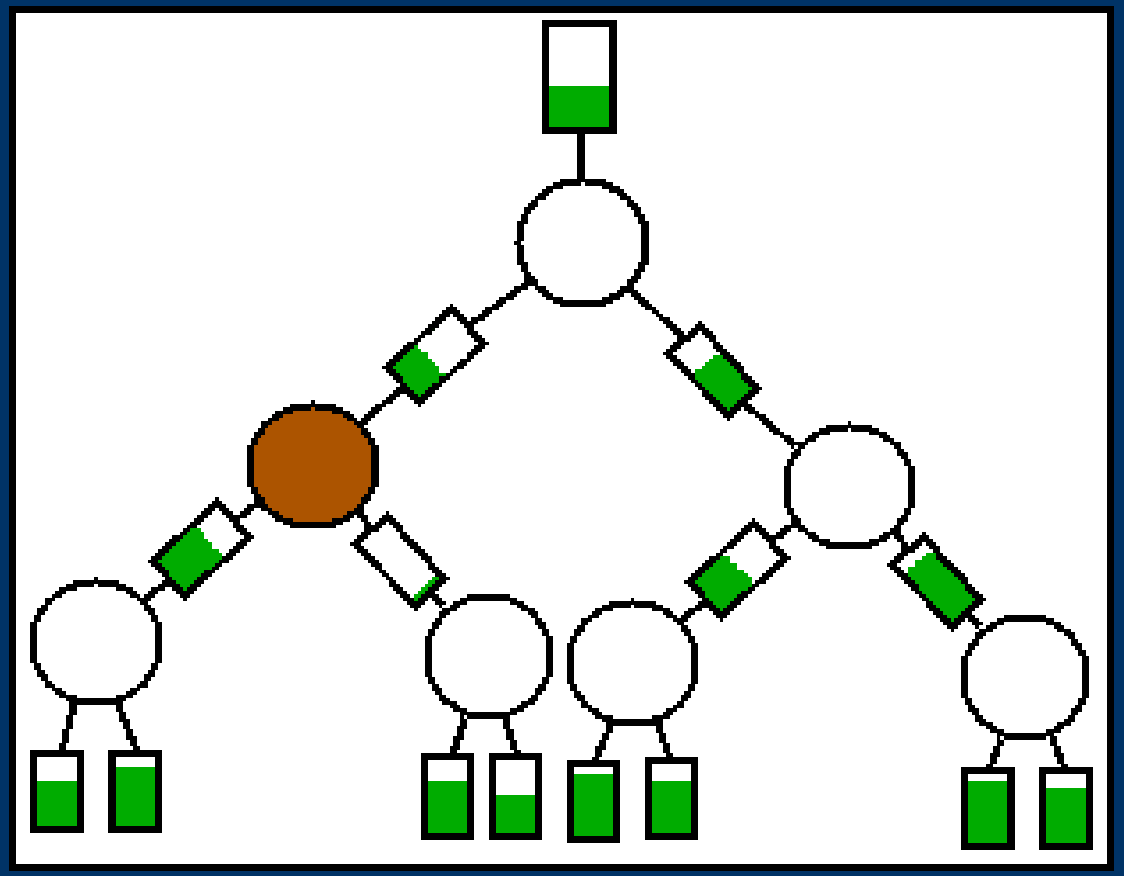

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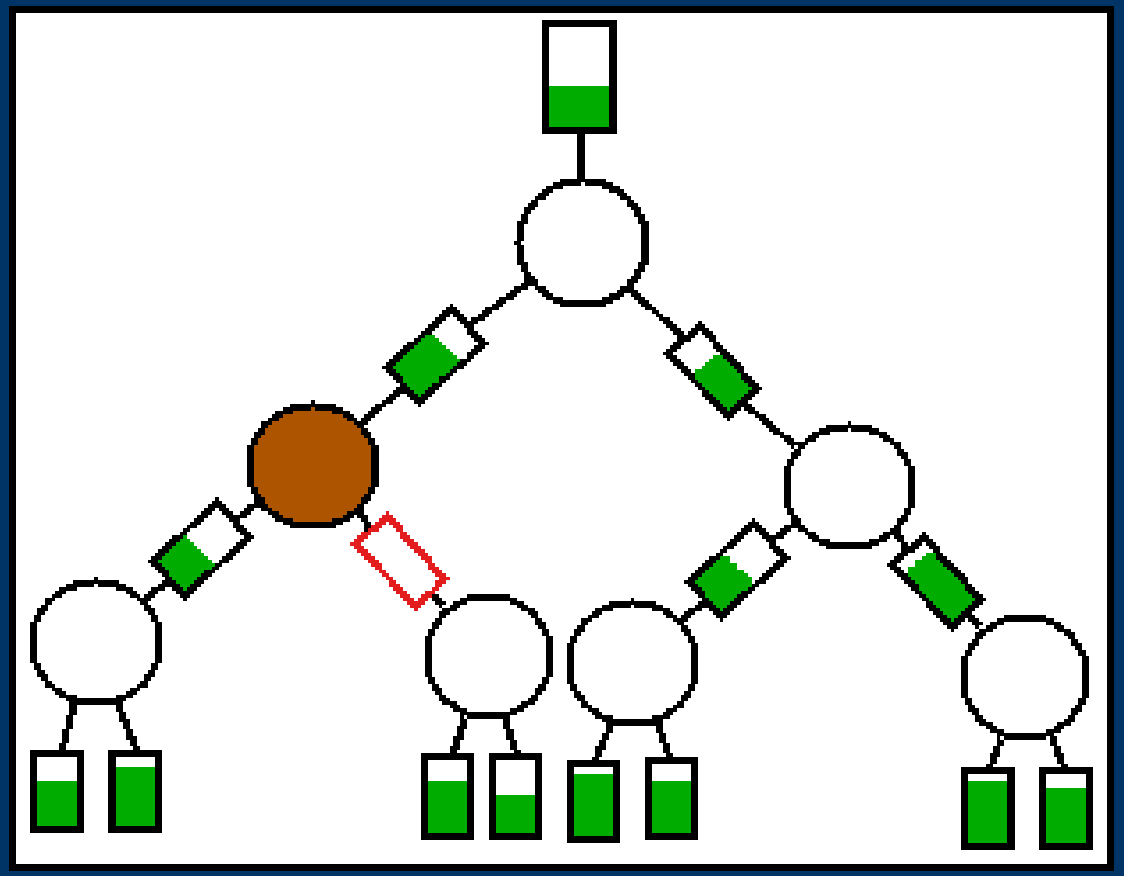

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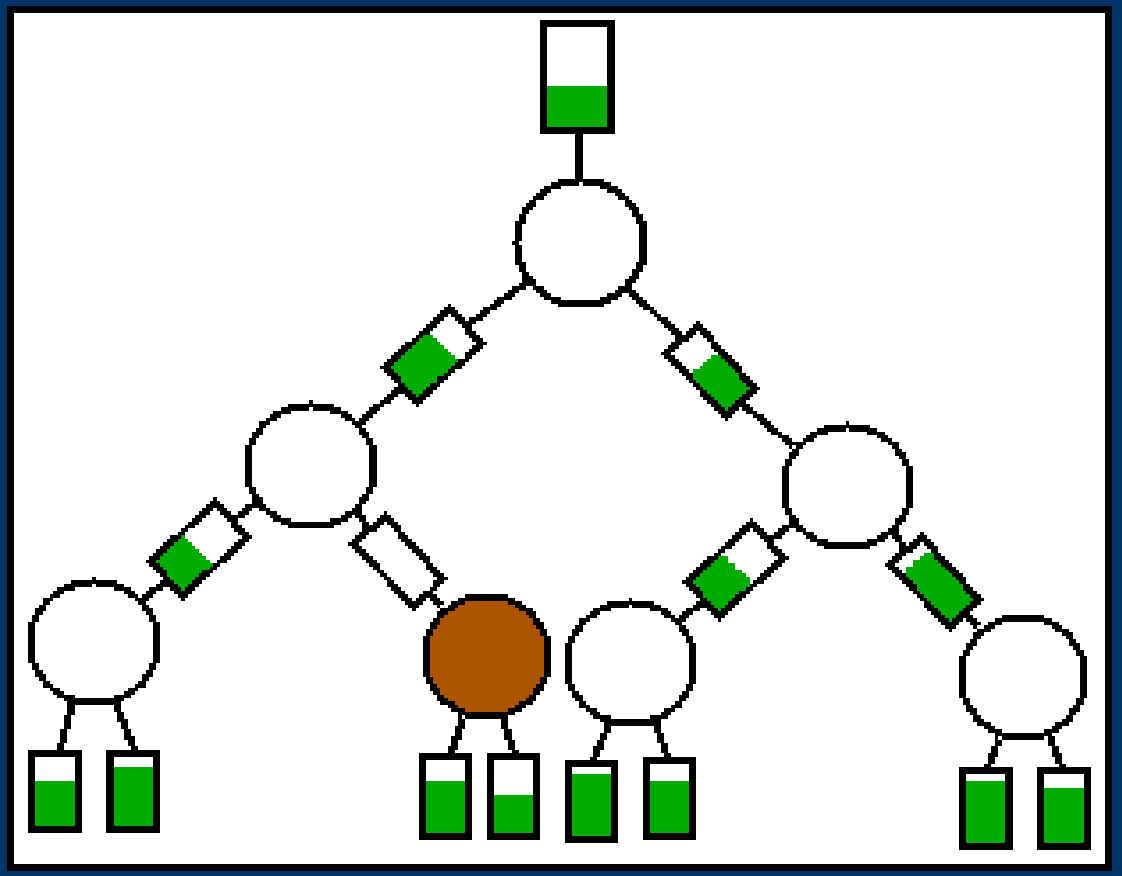

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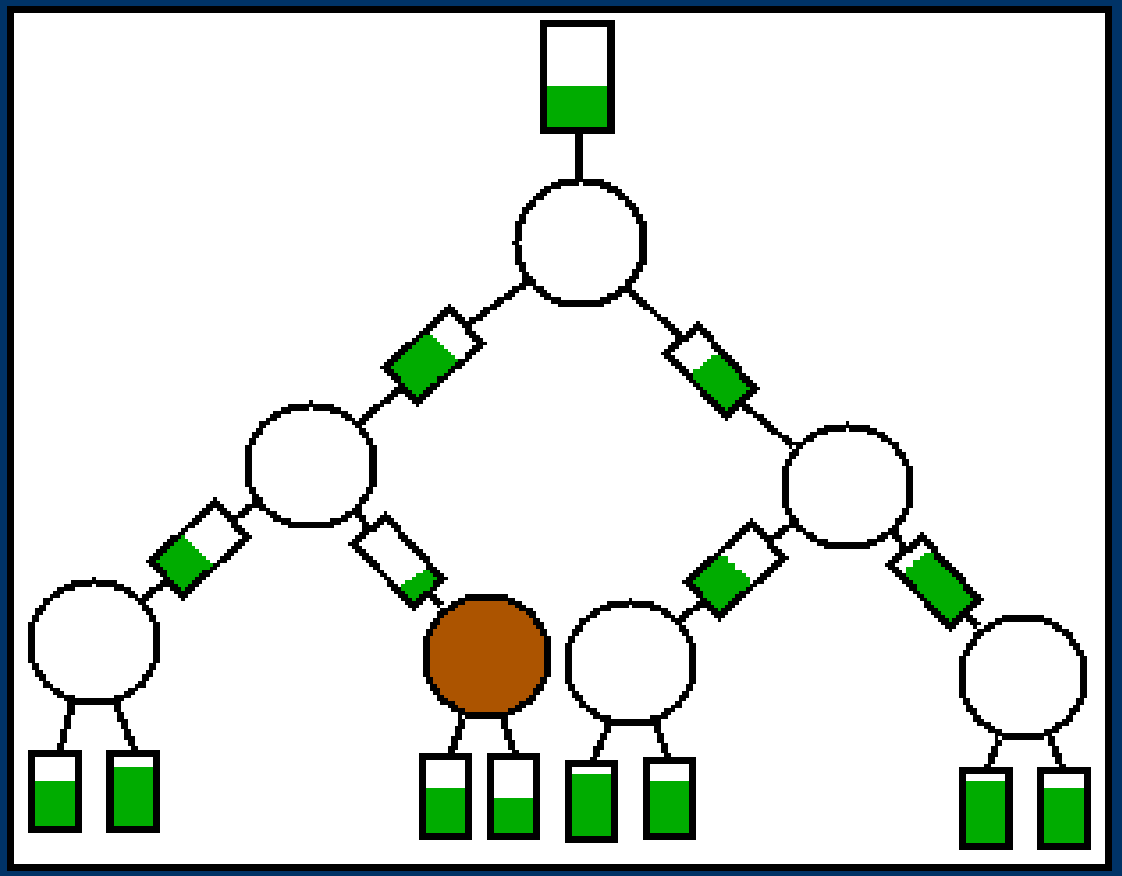

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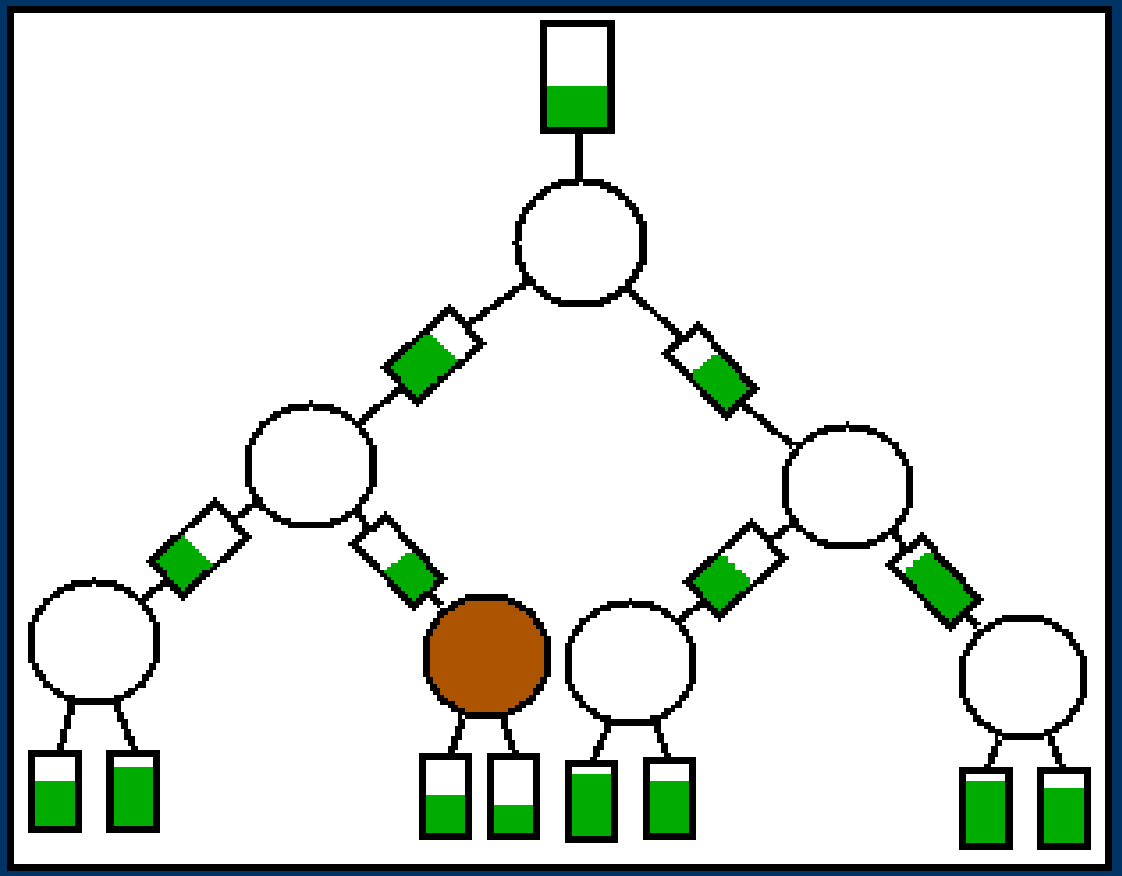

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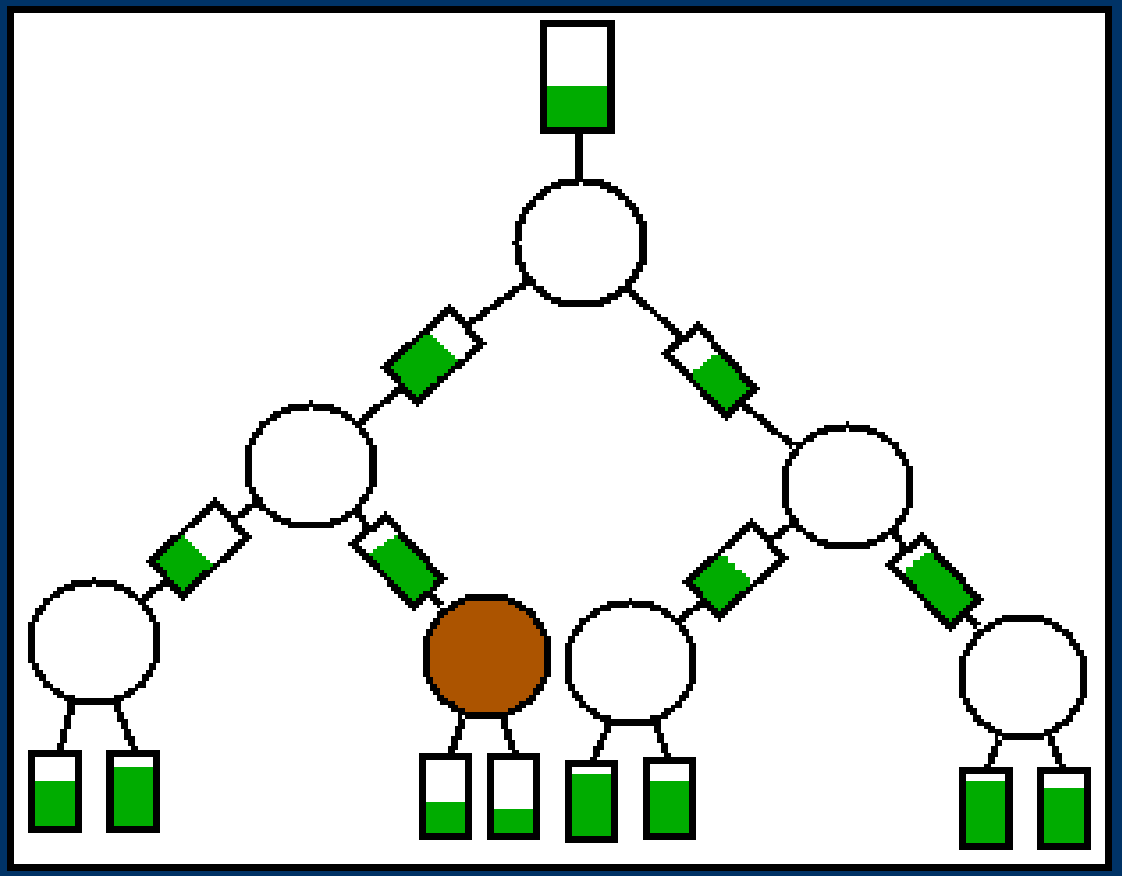

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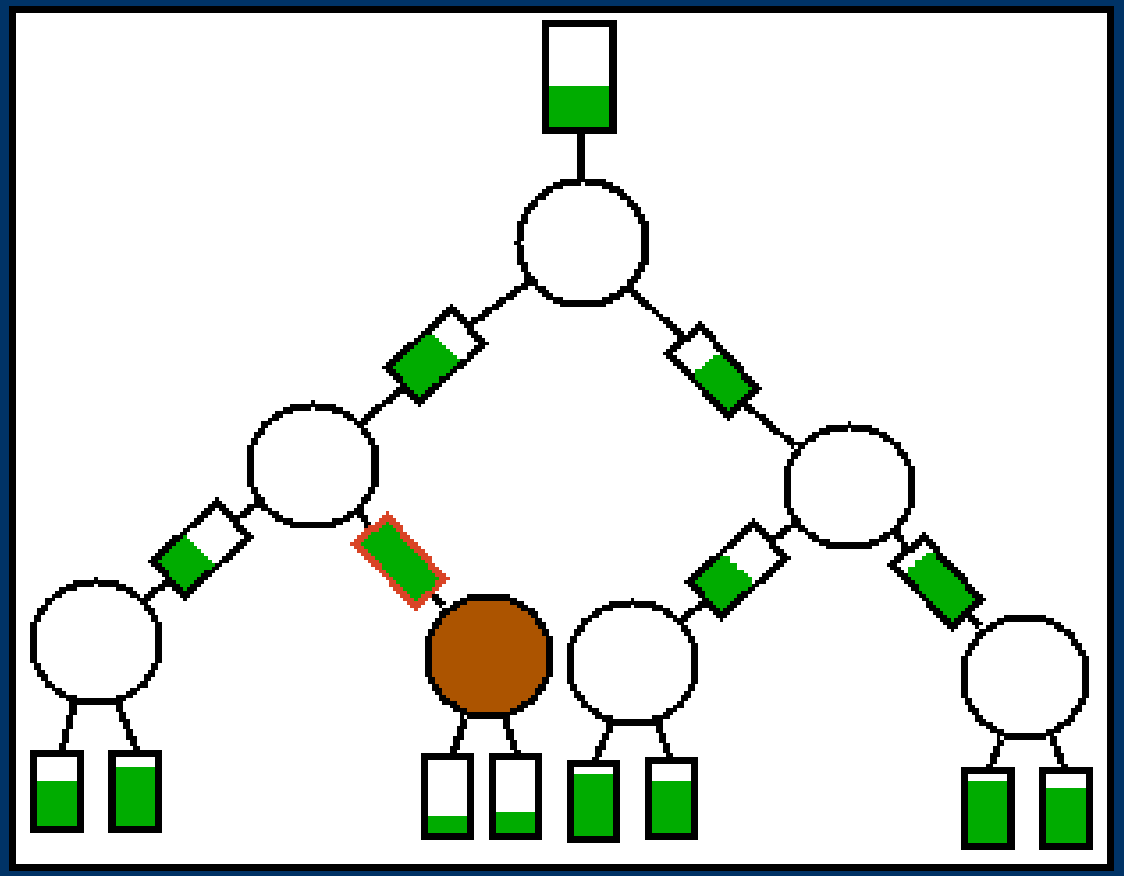

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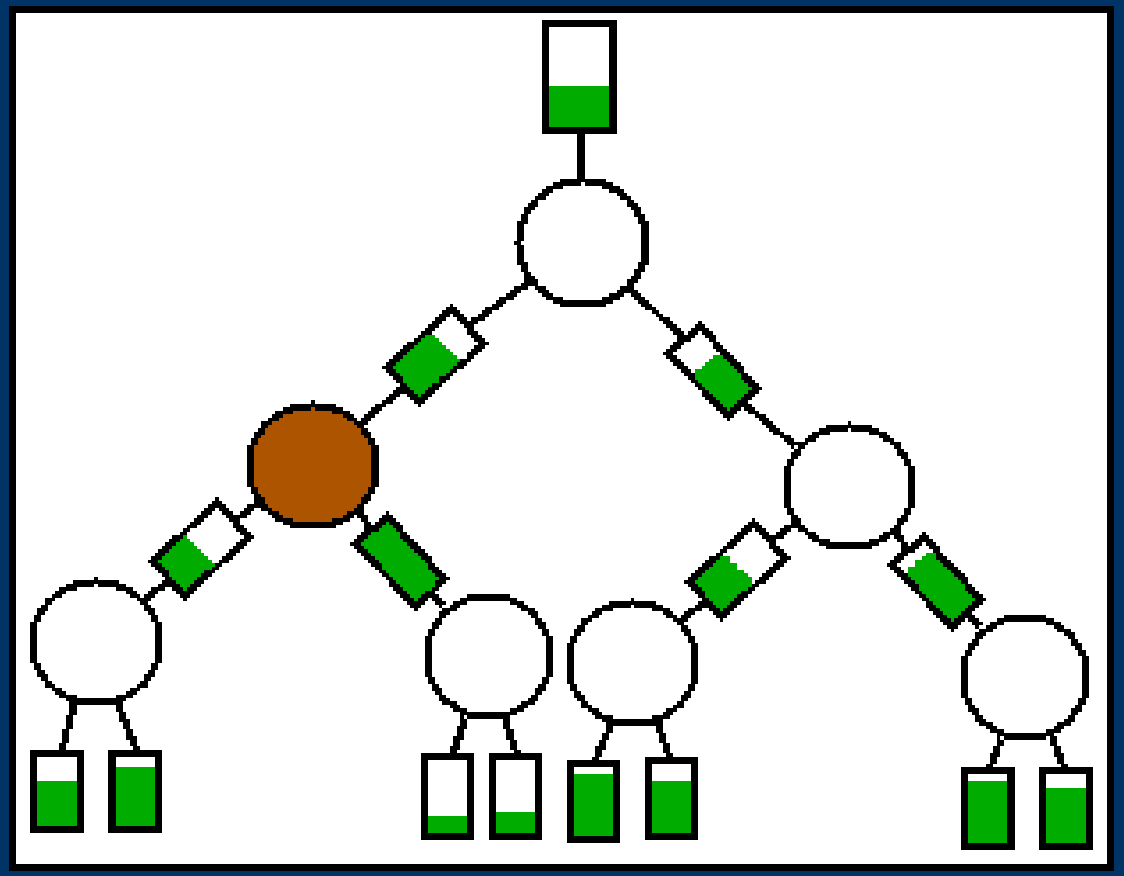
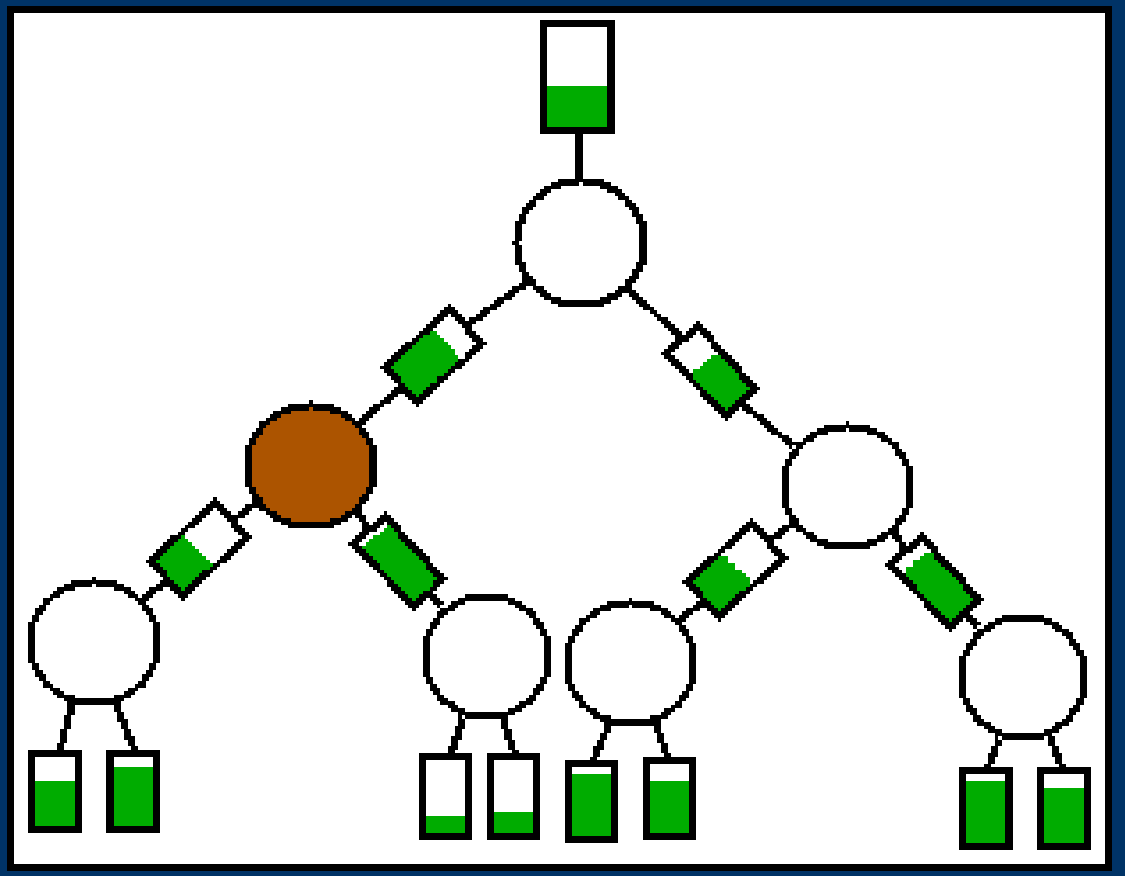
  - input is empty
    or
  - output is full



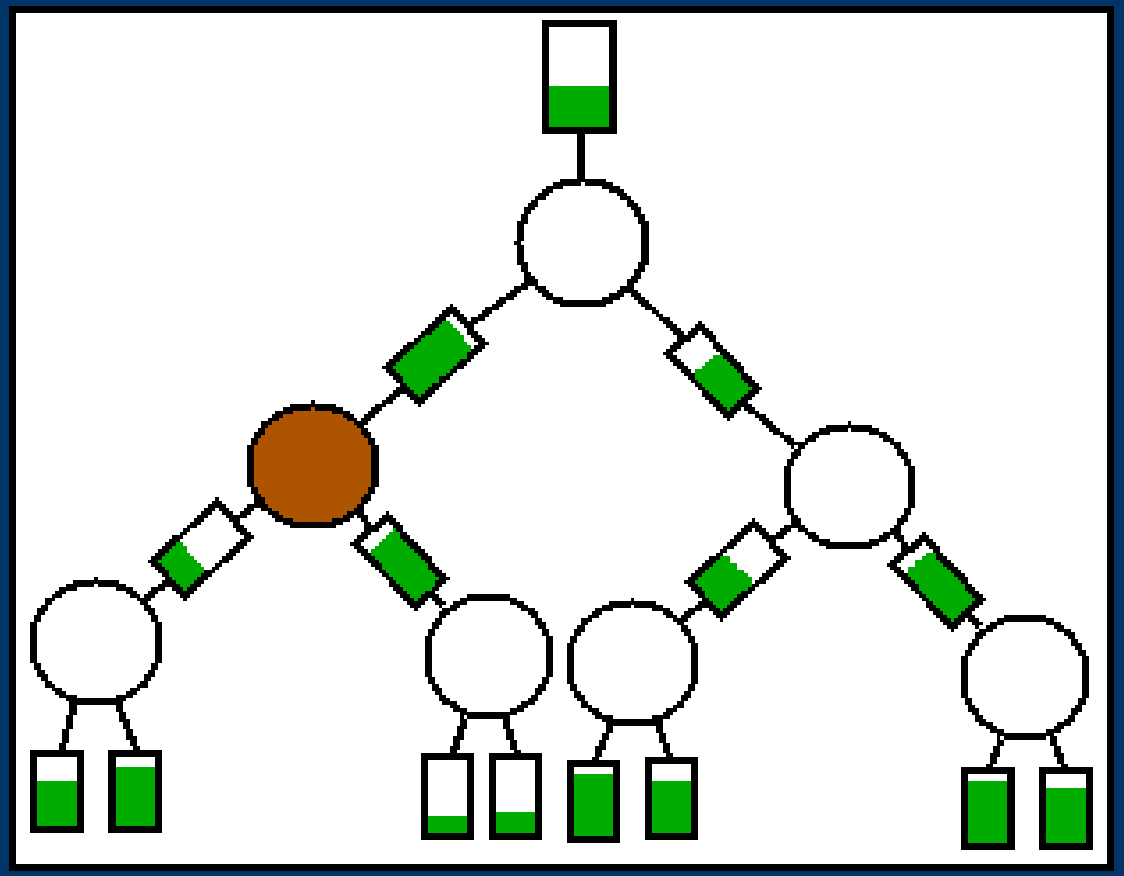
Funnelsort - k-merger

- Simple rules
- Start at the root
- Merge until:
  - input is empty
    or
  - output is full



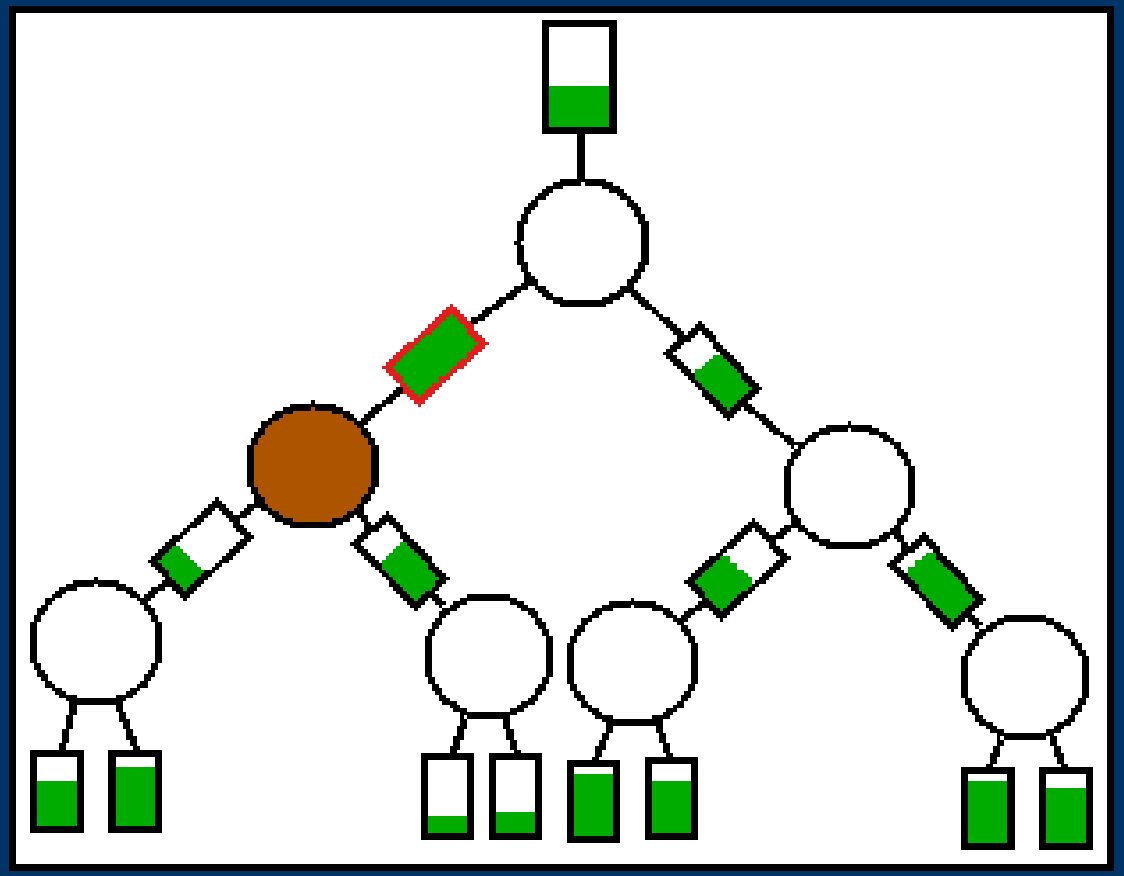
Funnelsort - k-merger

- Simple rules
- Start at the root
- Merge until:
  - input is empty
    or
  - output is full



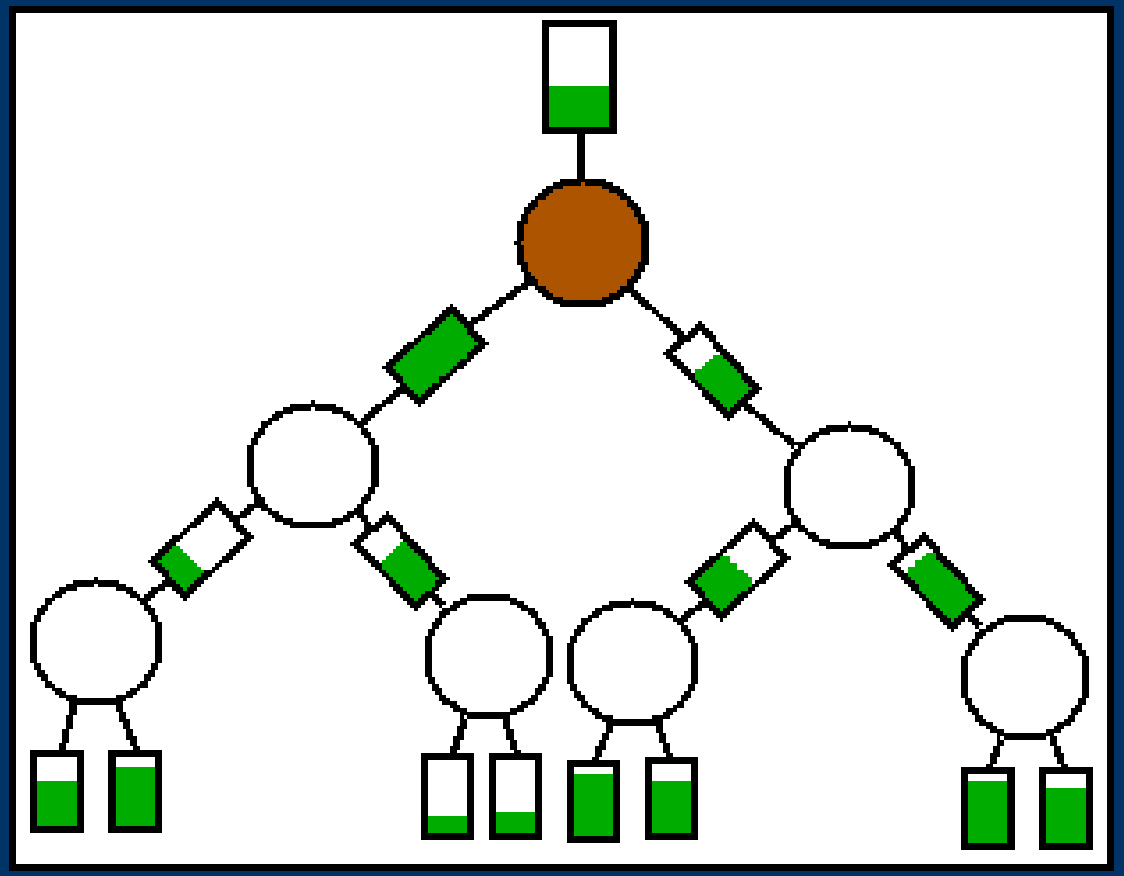
Funnelsort - k-merger

- Simple rules
- Start at the root
- Merge until:
  - input is empty
    or
  - output is full



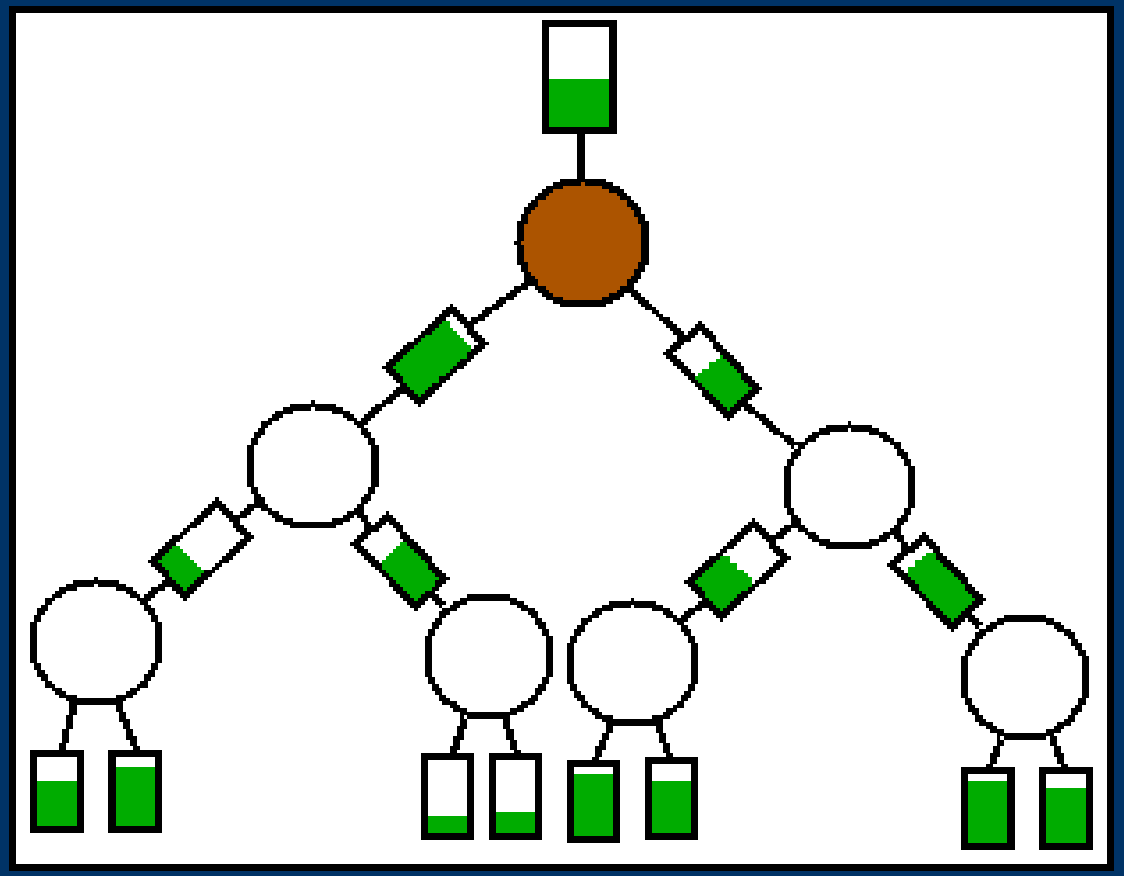
Funnelsort - k-merger

- Simple rules
- Start at the root
- Merge until:
  - input is empty
    or
  - output is full



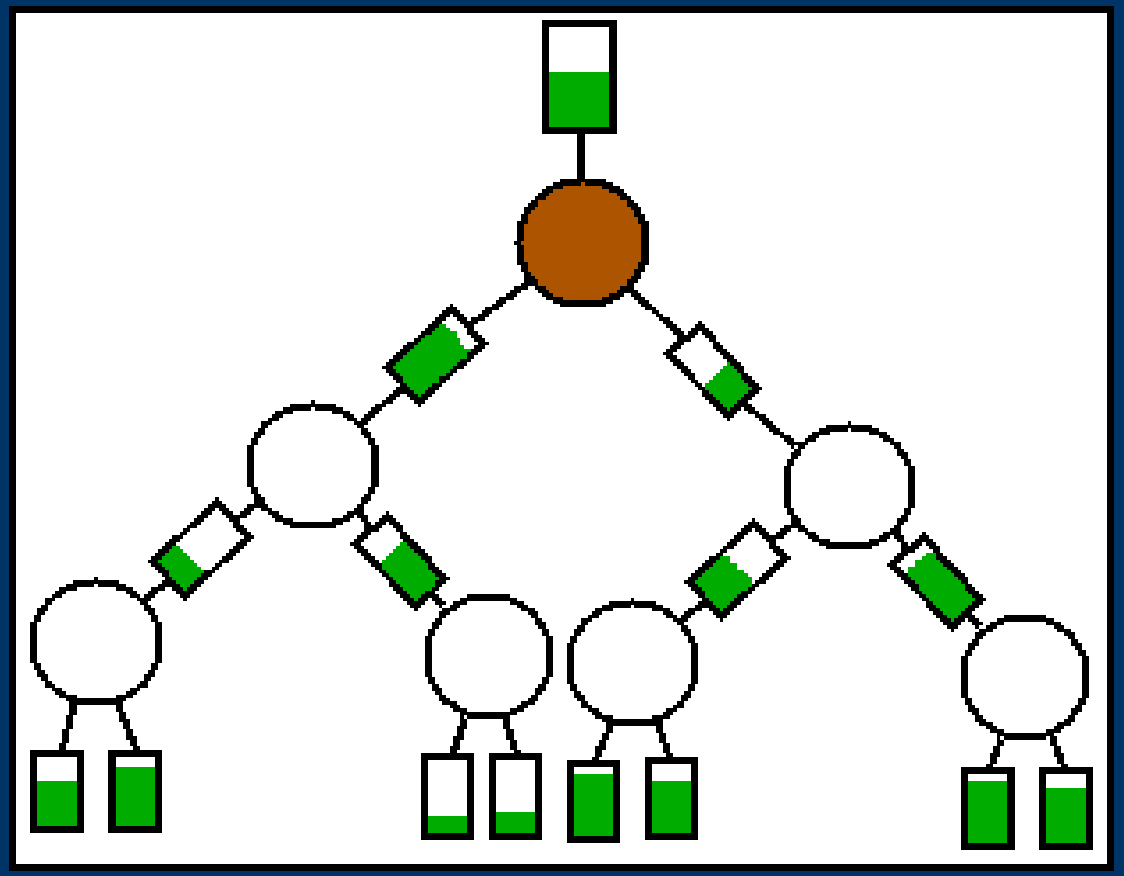
Funnelsort - k-merger

- Simple rules
- Start at the root
- Merge until:
  - input is empty
    or
  - output is full



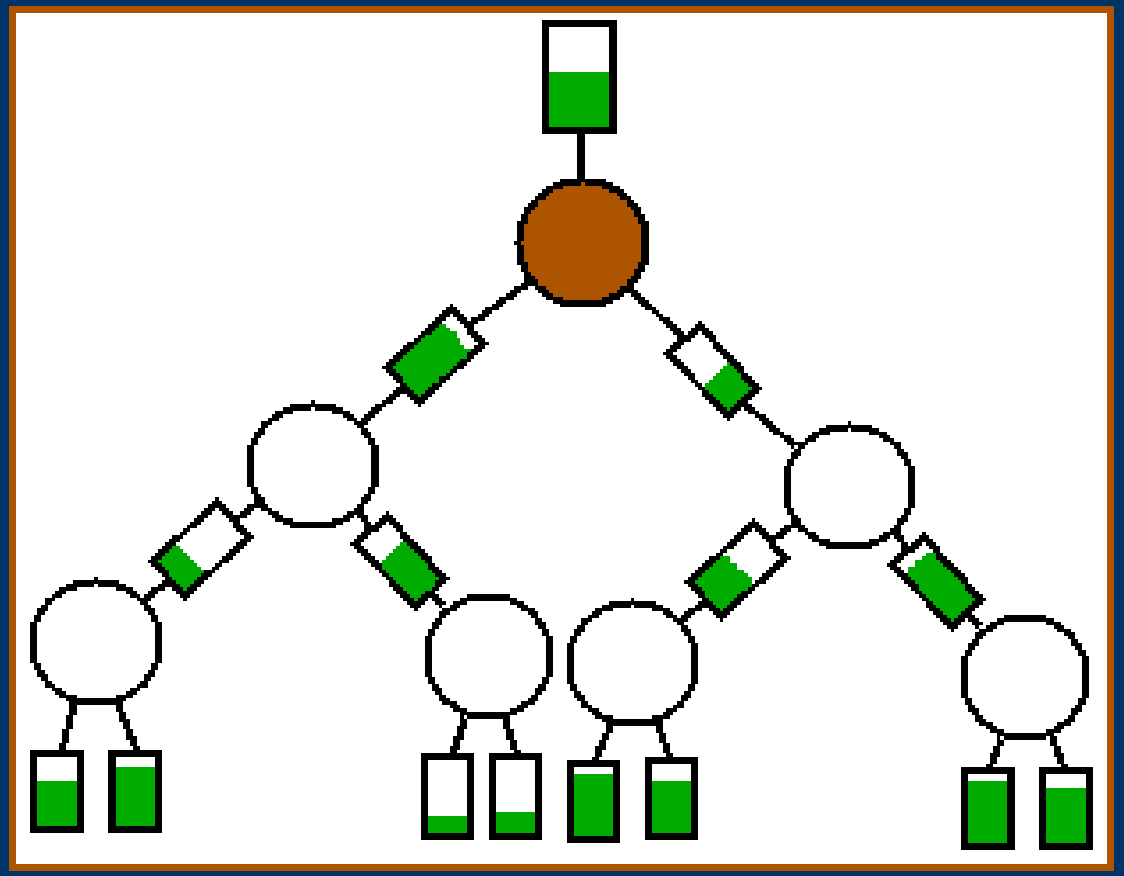
Funnelsort - k-merger

- Simple rules
- Start at the root
- Merge until:
  - input is empty
    or
  - output is full



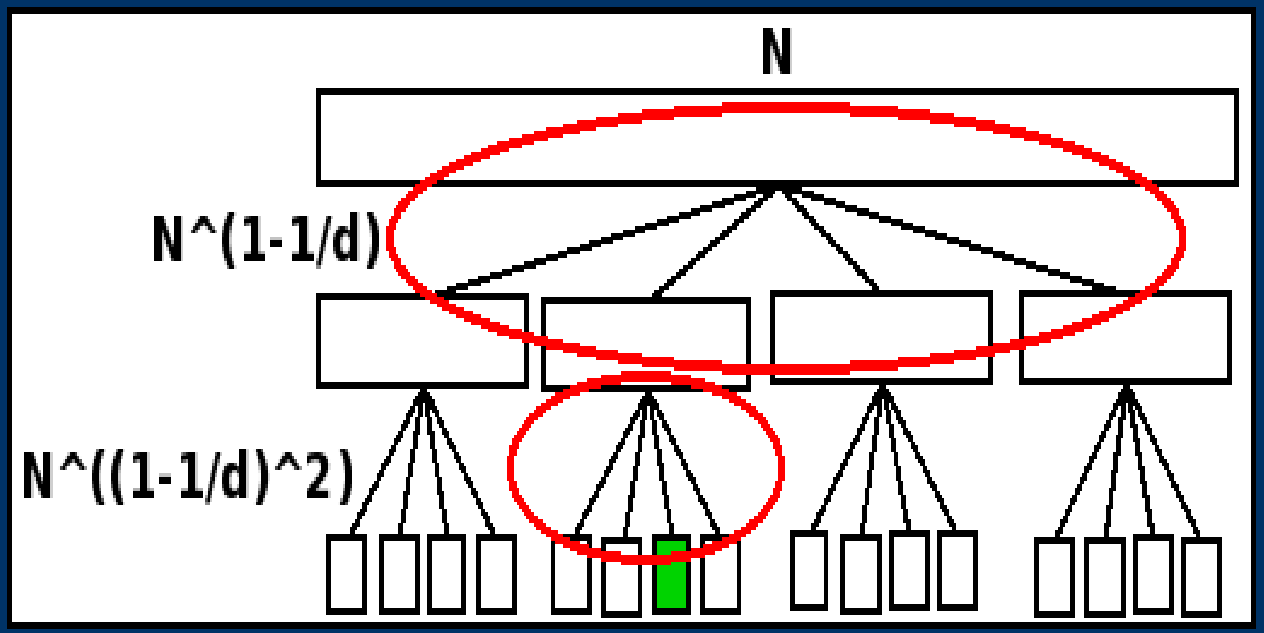
Funnelsort - k-merger

- And so on..
- Until input buffers are empty
- Output buffer is full



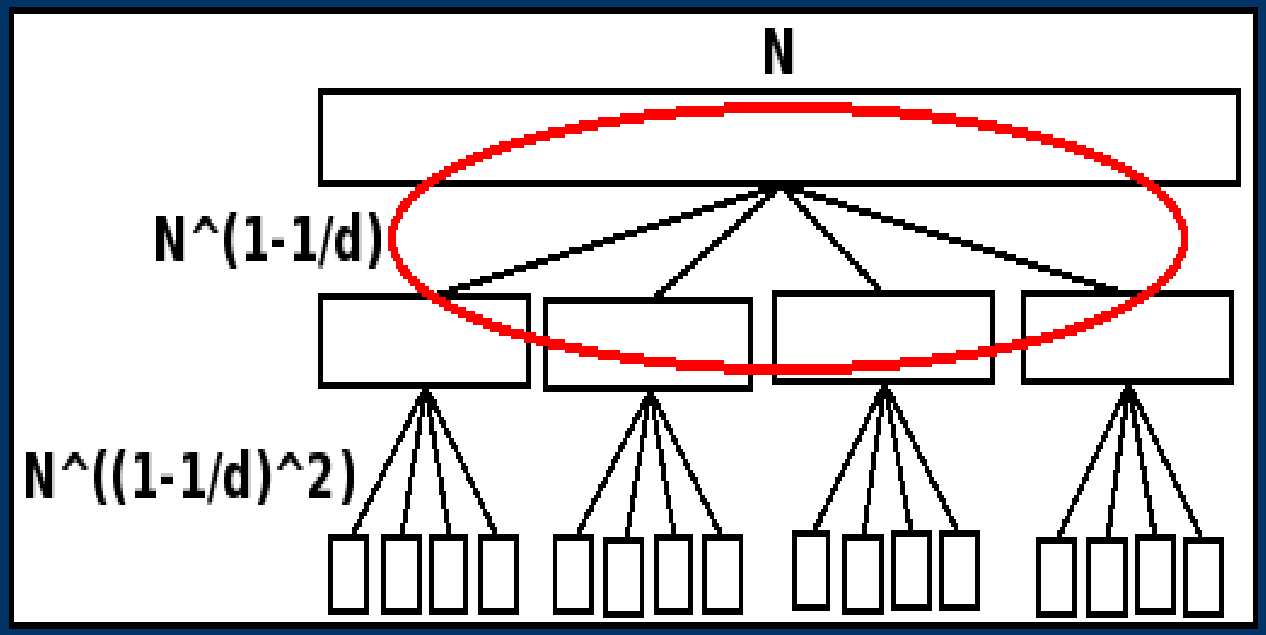
Funnelsort – I/O cost

- I/O cost to sort one element:
  - Sum of I/Os at each merge step



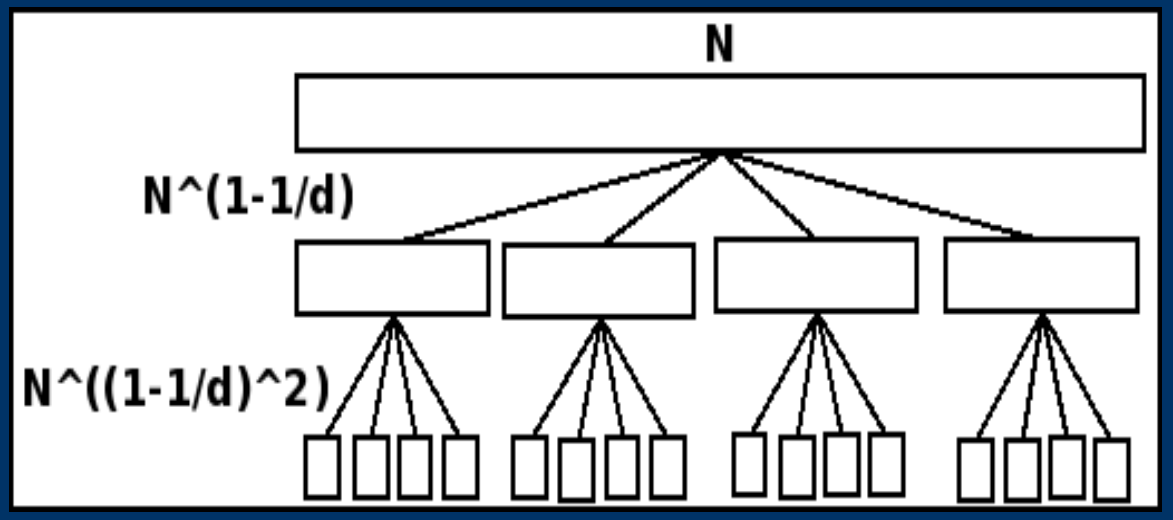
Funnelsort – I/O cost

- Using a k-merger (merging *m* elements)
  - Each element costs **O(log m / B)** I/O's
- *m* is the size of the inputs being merged



Funnelsort – I/O cost

- Total I/O cost to sort *one* element:
  - **O((log N) / B)** I/O's
- Total I/O cost to sort *N* elements:
  - **O((N log N) / B)** I/O's



Funnelsort – I/O cost

- I/O cost to sort one element:
  - Sum of I/Os at each merge step
- Total cost to sort 1 element is:

$$O((d \log n) / B)$$

Comparison

- Comparison with std:sort() from g++ 4.1.2 on Fedora 8
- Single quad-core processor
- 4KB CPU cache
- 8000MB memory (488 MB data set)

Comparison

- Sorting with a single thread
  - Quicksort is able to beat Funnelsort
  - By factor of 1.20
  - I/O isn't the largest factor
- Sorting with 4 threads
  - Funnelsort is faster than Quicksort
  - By factor of 1.06

Comparison

- With four cores competing:
  - Funnelsort improved relative to Quicksort by 22%
  - Funnelsort was able to outperform Quicksort

Comparison

- Fastest sorting methods
  - Using a bucket sort to merge results between processors

1)Funnelsort with 4 cores (5.10s)

2)Quicksort with 4 cores (5.59s)

3)Quicksort with 1 core (10.72s)

4)Funnelsort with 1 core (12.51s)

Conclusion

- When I/O access is limited, I/O efficiency becomes important
- Multi-core processors highlight the need for I/O efficient algorithm design
  - Even when data sets fit entirely inside main memory
- Utilizing multiple cores with I/O in mind provides the best solutions